GALACTIC MAGNETIC FIELDS, COSMIC RAYS AND WINDS
Part 1

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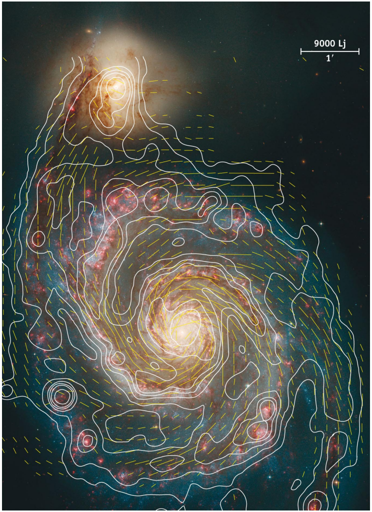
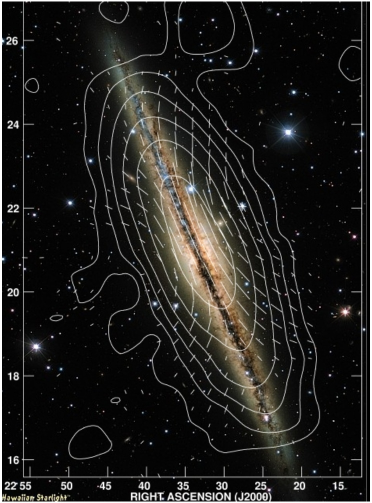
II Cosmology School, Kielce, 18.07.2016

M51



A. Fletcher et al. 2008

NGC891



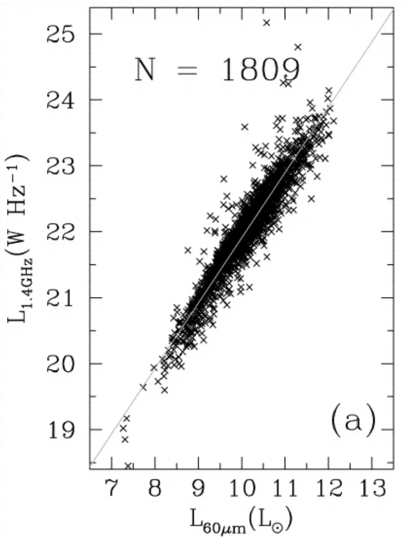
M. Krause et al. 2008

RADIO-FIR CORRELATION FOR A SAMPLE OF 1809 GALAXIES

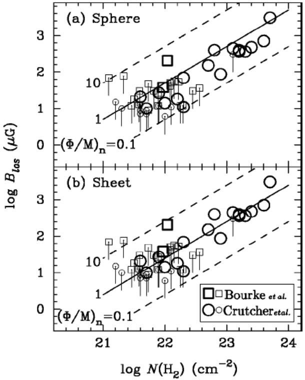
Yun, Reddy & Condon (2001)

Horizontal axis: FIR luminosity – indicator of star formation rate

Vertical axis: radio luminosity $L_{1.4GHz}$ – synchrotron emission of electrons in galactic magnetic fields.



All star-forming molecular clouds appear to be nearly critical: $\frac{M}{\Phi} \simeq \left(\frac{M}{\Phi}\right)_{crit}$

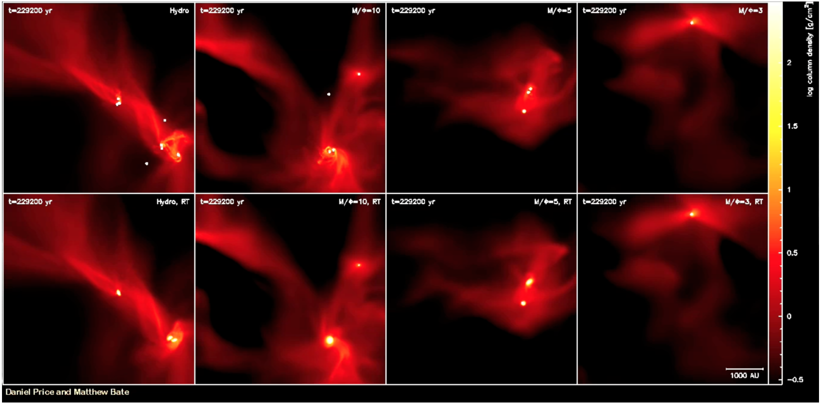


Bourke et al (2001)
Crutcher et al. (1999)

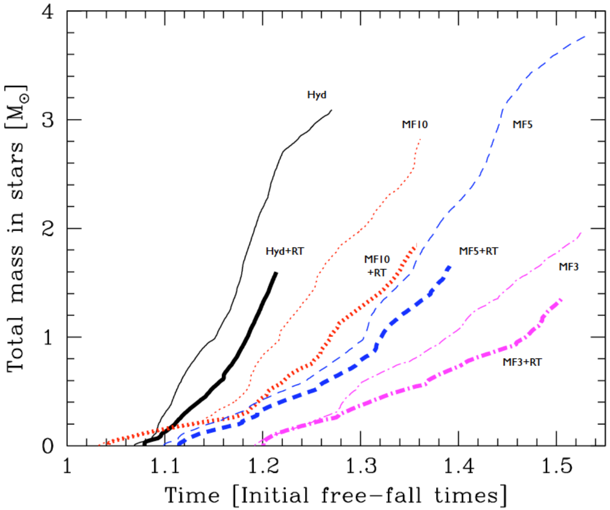
In presence of magnetic fields gravitational collapse is possible only if

$$\frac{M}{\Phi} > \left(\frac{M}{\Phi}\right)_{crit} \simeq 490\,\mathrm{g}\,\mathrm{G}^{-1} cm^{-2}$$

(Mestel & Spitzer 1956, Mouschovias & Spitzer 1976)



Gravitational collapse of a cloud depends on M/Φ (Price and Bate 2009)



STRONG MAGNETIC FIELD ⇒ REDUCED STAR FORMATION

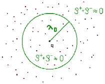
(Price and Bate 2009)

Recommended textbook: *The Physics of Fluids and Plasmas*, Arnab Rai Choudhuri, Cambridge University Press, 1998.

- Ionized gas, positive (i) and negative (e) charges move independently
- In a general case charge separation is possible: $\rho^+ \neq \rho^-$
- For relatively small distances (on 'microscopic' scales, typically meters for ISM)

$$r > \lambda_D = \left(\frac{k_B T}{8\pi n_o e^2} \right)^{\frac{1}{2}} \qquad \text{– DEBYE LENGTH}$$

from a point charge q the electric field of charge q is compensated by redistribution of positive and negative charges: **screening effect**



- On macroscopic scales charge separation can be neglected:

$$\rho_e = \rho^+ + \rho^- = 0$$

- Despite charge neutrality, electric current can appear due to electric field.
- When conducting medium is pervaded by electric field, then according to OHM's law

$$\vec{j'} = \sigma \vec{E'}$$

where $\vec{j'}$ – electric current density σ $\vec{E'}$ – electric field in a coordinate system comoving with fluid.
In the laboratory frame

$$\vec{j} = \vec{j'} + \rho_e \vec{v} = \vec{j'} \qquad (\rho_e = 0)$$

- Lorentz transformation of electromagnetic field:
  Denote: $\vec{E}$, $\vec{B}$ – in laboratory frame, $\vec{E}'$, $\vec{B}'$ – in a reference frame moving with velocity $\vec{v}$ (see e.g. Jackson's *Elektrodynamics*)

$$\vec{E}_{\parallel}{}' = \vec{E}_{\parallel}, \qquad \vec{B}_{\parallel}{}' = \vec{B}_{\parallel}$$

$$\vec{E}_{\perp}{}' = \frac{\vec{E}_{\perp} + \frac{\vec{v}}{c} \times \vec{B}_{\perp}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \vec{B}_{\perp}{}' = \frac{\vec{B}_{\perp} - \frac{\vec{v}}{c} \times \vec{E}_{\perp}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

  where $\|$ i $\perp$ denotes respectively parallel and perpendicular to $\vec{v}$.
- When $\frac{v^2}{c^2} \ll 1$ (non-relativistic fluid motion)

$$\vec{E}' \simeq \vec{E} + \frac{\vec{v}}{c} \times \vec{B},$$

  and

$$\vec{j} = \sigma \left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right) \quad \text{OHM'S LAW}$$

- Maxwell's equations in CGS units

$$\nabla \cdot \vec{D} = 4\pi \rho_e \tag{1}$$

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \tag{2}$$

$$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \tag{3}$$

$$\nabla \cdot \vec{B} = 0 \tag{4}$$

- $D = \epsilon E$ and $B = \mu H$. For a monoatomic gas dielectric permittivity $\epsilon = 1$, and pearmeability $\mu = 1$, therefore

$$\vec{D} = \vec{E}, \qquad \vec{B} = \vec{H}$$

- We assume that L and τ are the typical length scales and time scales of the system, $v \simeq L/\tau$ is the typical velocity

$$\frac{\left|\frac{1}{c}\frac{\partial \vec{D}}{\partial t}\right|}{\left|\nabla \times \vec{H}\right|} \simeq \frac{\frac{|\vec{D}|}{c\tau}}{\frac{|\vec{H}|}{L}} \simeq \frac{v}{c}\frac{|\vec{E}|}{|\vec{B}|}$$

- Similarly, from Faraday's induction law

$$1 \simeq \frac{\left|\nabla \times \vec{E}\right|}{\left|\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\right|} \simeq \frac{\frac{|\vec{E}|}{L}}{\frac{|\vec{B}|}{c\tau}} \simeq \frac{\left|\vec{E}\right|}{\left|\vec{B}\right|}\frac{c}{v} \Rightarrow \frac{\left|\vec{E}\right|}{\left|\vec{B}\right|} \simeq \frac{v}{c},$$

thus

$$\frac{\left|\frac{1}{c}\frac{\partial \vec{D}}{\partial t}\right|}{\left|\nabla \times \vec{H}\right|} \simeq \frac{v^2}{c^2} \ll 1.$$

Therefore, in the nonrelativistic case equation (2) reduces to

$$\nabla \times \vec{B} = \frac{4\pi}{c} j \qquad \text{(Ampere's law)} \qquad \text{(2a)}$$

We shall derive the equation describing evolution of magnetic field in a moving ionised medium.

- From Faraday's induction law:

$$(3) \Rightarrow \frac{\partial \vec{B}}{\partial t} = -c\ \nabla \times \vec{E}$$

- From Ohm's law:

$$\vec{E} = \frac{\vec{j}}{\sigma} - \frac{\vec{v}}{c} \times \vec{B}$$

$$\nabla \times \vec{E} = \nabla \times \left(\frac{\vec{j}}{\sigma}\right) - \nabla \times \left(\frac{\vec{v}}{c} \times \vec{B}\right)$$

- From Ampere's law:

$$(2a) \Rightarrow \vec{j} = \frac{c}{4\pi}(\nabla \times \vec{B})$$

Therefore

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \frac{c^2}{4\pi\sigma} \underbrace{\nabla \times (\nabla \times \vec{B})}_{\nabla \underbrace{(\nabla \cdot \vec{B})}_{=\mathbf{0}} - \Delta \vec{B}}$$

We obtain **INDUCTION EQUATION** describing magnetic field evolution in a medium of finite electric conductivity

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \lambda \Delta B,$$

where $\lambda \equiv \frac{c^2}{4\pi\sigma}$ is the **magnetic diffusivity coefficient**.
Assume that B, V, L represent typical values of magnetic induction, velocity and length scale in the system.

$$\nabla \times (\vec{v} \times \vec{B}) \sim \frac{VB}{L}$$

$$\lambda \nabla^2 \vec{B} \sim \frac{\lambda B}{L^2}$$

Magnetic Reynolds number

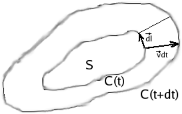
$$\mathbf{Re_M} = \frac{VB/L}{\lambda B/L^2} = \frac{LV}{\lambda}$$

$\mathbf{Re_M}$ measures significance of magnetic diffusive transport with respect to advective transport of magnetic field.

Magnetic diffusion is negligible when $Re_M \gg 1$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B})$$ the limit of ideal MHD

Consider contour C encircling surface S moving with the fluid



$d\vec{l}$ - element konturu C

The change of surface area within contour C in time interval dt: $\int_C \vec{v} \times d\vec{l} dt$. The rate of change of magnetic flux threading surface S

$$\frac{d}{dt}\int_S \vec{B} \cdot \underbrace{d\vec{S}}_{\text{surface element}} = \underbrace{\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}}_{\text{change of } \vec{B}} + \underbrace{\int_C \vec{B} \cdot (\vec{v} \times d\vec{l})}_{\text{displacement of contour C}}$$

From the identity

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

we get

$$\int_C \vec{B}\cdot(\vec{v}\times\vec{dl}) = \int_C -(\vec{v}\times\vec{B})\cdot\vec{dl}$$

From Stokes theorem

$$\int_C -(\vec{v}\times\vec{B})\cdot\vec{dl} = -\int_S \nabla\times(\vec{v}\times\vec{B})\cdot d\vec{S}$$

Therefore

$$\frac{d}{dt}\int \vec{B}\cdot d\vec{S} = \int_S \left(\frac{\partial\vec{B}}{\partial t} - \nabla\times(\vec{v}\times\vec{B})\right)\cdot d\vec{S} = 0$$

The expression under the last integral vanishes, according to induction equation in the ideal MHD limit ($\lambda = 0$)

Magnetic flux threading surface S encircled by contour C comoving with the fluid is constant.

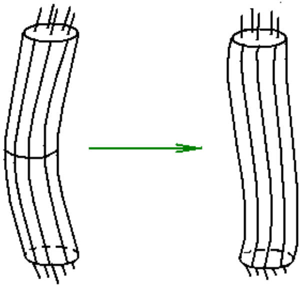
$\Rightarrow$ this property is named MAGNETIC FIELD FREEZING

MAGNETIC FLUX TUBE:

– cyllindrical region formed by a bunch of magnetic field lines

MAGNETIC FIELD FREEZING:

In the limit of ideal MHD magnetic field is advected with the fluid, fluid elements belong always to the same flux tube.



MAGNETIC DIFFUSION

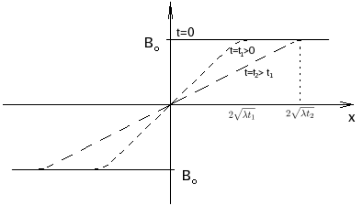
When magnetic diffusion dominates ($Re_M \ll 1$), induction equation reduces to the diffusion equation

$$\frac{\partial \vec{B}}{\partial t} = \lambda \nabla^2 \vec{B}, \qquad \text{in 1-D:} \qquad \frac{\partial B}{\partial t} = \lambda \frac{\partial^2 B}{\partial x^2}$$

Time-dependent solution:

Consider a 1-D configuration:
Assume, initial $\vec{B} \parallel y$

$$B(x,0) = \begin{cases} B_0 \text{ dla } x > 0 \\ -B_0 \text{ dla } x < 0 \end{cases}$$



$$\text{Magnetic flux:} \int_{-\infty}^{\infty} B dx = \text{const}$$

$$\text{Magnetic energy:} \int_{-\infty}^{\infty} \frac{B^2}{8\pi} dx : \text{decays.}$$

The rate of change of magnetic energy:

$$\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\frac{B^2}{8\pi}dx = \int_{-\infty}^{\infty}\frac{B}{4\pi}\frac{\partial B}{\partial t}dx = \int_{-\infty}^{\infty}\frac{B}{4\pi}\lambda\frac{\partial^2 B}{\partial x^2}dx$$

Integration by parts and substitution of j from Amper's law

$$\frac{\partial B}{\partial x} = \frac{4\pi}{c}j,$$

lead to

$$\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\frac{B^2}{8\pi}dx = -\int_{-\infty}^{\infty}\underbrace{\frac{4\pi\lambda}{c^2}}_{=\frac{1}{\sigma}}j^2 dx.$$

Therefore

$$\frac{\partial}{\partial t}\frac{B^2}{8\pi} = \frac{-j^2}{\sigma}$$

MAGNETIC ENERGY LOSS THROUGH ELECTRIC CURRENT DISSIPATION: **'OHMIC HEATING'**

$\Rightarrow$ **production rate of thermal energy** $= \frac{j^2}{\sigma}$

In the presence of magnetic field the equation of motion of ionised gas includes Lorentz force

$$\vec{F}_L = \rho_e \vec{E} + \frac{1}{c}\vec{j} \times \vec{B}$$

where $\rho_e(=0)$ net density of electric charges,
$\vec{j} = ne(\vec{v_i} - \vec{v_e})$ - electric current density
Gas equation of motion: Euler equation with magnetic forces

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{\vec{F}}{m} - \frac{1}{\rho}\nabla p + \frac{1}{c}(\vec{j} \times \vec{B})$$

Lorentz force:

$$\begin{aligned}
\vec{F}_{\mathrm{L}} &= \frac{1}{c}(\vec{j} \times \vec{B}) = \\
&= \frac{1}{4\pi}(\vec{\nabla} \times \vec{B}) \times \vec{B} = \\
&= \nabla\left(\frac{B^2}{8\pi}\right) + \frac{(\vec{B} \cdot \vec{\nabla})\vec{B}}{4\pi}
\end{aligned}$$

EQUATION OD MOTION of ionised gas in presence of magnetic field

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{\vec{F}}{m} - \frac{1}{\rho}\nabla\left(p + \frac{B^2}{8\pi}\right) + \frac{(\vec{B} \cdot \vec{\nabla})\vec{B}}{4\pi\rho}$$

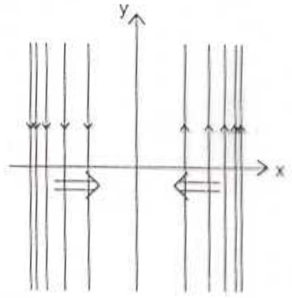
The quantity $\frac{B^2}{8\pi}$ is the magnetic energy density. In ionised medium it acts also as an additional pressure component: **MAGNETIC PRESSURE**.

The second magnetic term:

$$\frac{1}{4\pi}(\vec{B} \cdot \nabla)\vec{B} = \frac{B^2}{4\pi}\frac{\vec{n}}{R}$$

represents a force that is perpendicular to magnetic field ($\vec{n}$) and inversely proportional to the curvature radius of magnetic field line (R):
MAGNETIC TENSION FORCE

EXAMPLES:

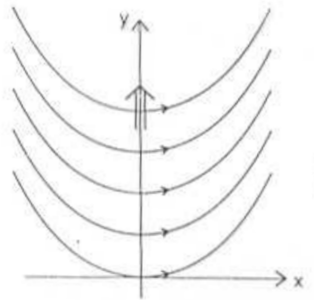


(a)

$$\vec{B} = x\hat{e}_y, \, \vec{j} = \frac{c}{4\pi}\nabla \times B = \frac{c}{4\pi}\frac{\partial B_y}{\partial x}\hat{e}_z = \frac{c}{4\pi}\hat{e}_z$$

$$\frac{1}{4\pi}(\vec{B}\cdot\nabla)\vec{B} = \frac{B_y}{4\pi}\frac{\partial B_y}{\partial y}\hat{e}_y = 0$$

$$-\nabla\left(\frac{B^2}{8\pi}\right) = -\nabla\left(\frac{x^2}{8\pi}\right) = -\frac{x}{4\pi}\hat{e}_x$$



(b)

$$\vec{B} = \hat{e}_x + x\hat{e}_y$$

$$\text{field lines: } \frac{dy}{dx} = x \Rightarrow y = \frac{1}{2}x^2 + c$$

$$\frac{1}{4\pi}(\vec{B}\cdot\nabla)\vec{B} = \frac{1}{4\pi}\left(\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}\right)(\hat{e}_x + x\hat{e}_y) = \frac{\hat{e}_y}{4\pi}$$

$$-\nabla\left(\frac{B^2}{8\pi}\right) = -\nabla\left(\frac{1+x^2}{8\pi}\right) = -\frac{x}{4\pi}\hat{e}_x$$

$$= 0 \text{ on y axis}$$

FULL SET OF MHD EQUATIONS

Equation of motion, continuity equation, energy equation (ϵ - thermal energy per unit mass):

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla\Phi - \frac{1}{\rho}\nabla\left(p + \frac{B^2}{8\pi}\right) + \frac{(\vec{B} \cdot \nabla)\vec{B}}{4\pi\rho}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\vec{v}) = 0$$

$$\rho\left(\frac{\partial \epsilon}{\partial t} + \vec{v} \cdot \nabla\epsilon\right) + p\nabla\vec{v} = \eta j^2$$

Induction equation, divergence-free condition for magnetic field, Ampere's law:

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{c^2\eta}{4\pi}\nabla^2\vec{B}$$

$$\nabla \cdot \vec{B} = 0$$

$$\vec{j} = \frac{c}{4\pi}\nabla \times \vec{B}$$

Equation of state: $p\rho^{-\gamma} = \text{ const } \iff p = (\gamma - 1)\rho\epsilon, \qquad \gamma = \frac{C_P}{C_V}$

Alfven waves

Phase speed:

$$\frac{\omega}{k} = v_A \equiv \frac{B_0}{\sqrt{4\pi\rho_0}} \qquad \text{Alfvén speed}$$

Restoring force: magnetic tension.

Interpretation: Waves due to transversal bending of magnetic field lines.

Fast and slow magnetosonic waves

Phase speeds:

$$\frac{\omega_1^2}{k^2} = \frac{1}{2}\left[(c_s^2 + v_A^2) - \left((c_s^2 + v_A^2)^2 - 4c_s^2 v_A^2 \cos^2\theta\right)^{1/2}\right]$$

$$\frac{\omega_2^2}{k^2} = \frac{1}{2}\left[(c_s^2 + v_A^2) + \left((c_s^2 + v_A^2)^2 - 4c_s^2 v_A^2 \cos^2\theta\right)^{1/2}\right]$$

Restoring forces: gas pressure and magnetic pressure.

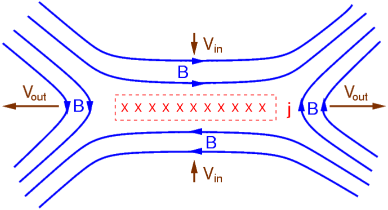
Interpretation: waves due to compression of gas and magnetic field

The fast wave: gas pressure and magnetic pressure in phase

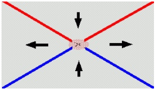
The slow wave: gas pressure and magnetic pressure in counter phase

Tangent discontinuities form in magnetic field spontaneously

Parker(1957), Sweet (1958), Petchek (1964)



$$\vec{j} = \frac{c}{4\pi}\nabla \times \vec{B}$$

- Ohmic dissipation of current $\vec{j} = \frac{c}{4\pi}\nabla \times \vec{B}$ is associated with annihilation of opposite magnetic fields.
- Magnetic energy is converted into thermal and kinetic energy.
- Magnetic field topology changes: 
- Reconnected lines are advected out of the current sheet.

- The Sweet-Parker model is inefficient. Reconnection rate (inflow velocity)

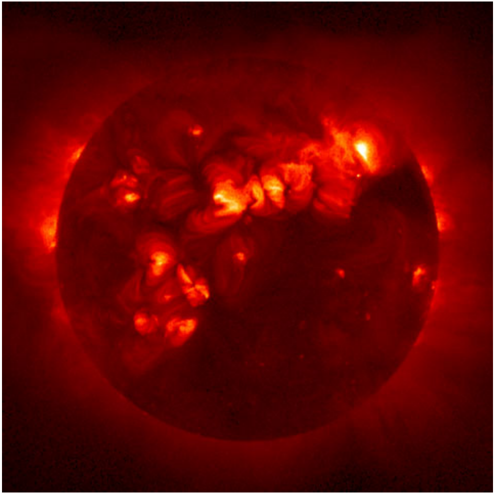
$$v_{in} \approx \frac{v_A}{\sqrt{\mathrm{Re}_M}}$$

can be very small in astrophysical conditions ($\mathrm{Re_M} \sim 10^{10} \div 10^{18}$)

- Models of **fast magnetic reconnection** (Petchek 1964), and models of **turbulent reconnection** (Kowal et al. 2009) lead to

$$v_{in} \approx \frac{v_A}{\ln \mathrm{Re}_M}$$

The reconnection rates can be as high as a few % of v_A.



Sun in X-rays (TRACE) – plasma at 3-6 MK

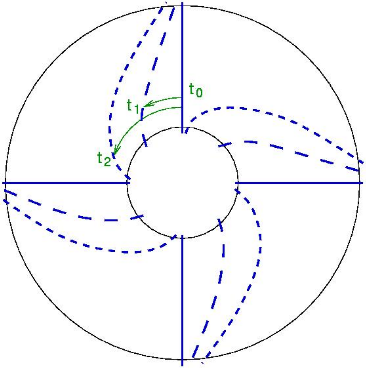
Fast magnetic reconnection is commonly accepted as the mechanism responsible for heating of Solar corona.

- Initial magnetic field in young galaxies
  can be generated in stars and scattered in the ISM by SNe
  $\Rightarrow$Mean magnetic field in the galactic scale (Rees, 1987, 1994)
  $$B \sim 10^{-9}\mathrm{G}$$
- Contemporary magnetic fields in spiral galaxies
  $$B \sim 3 \div 20 \times 10^{-6}\mathrm{G}$$
- EFFICIENT MAGNETIC FIELD AMPLIFICATION IS NECESSARY
- Amplification of magnetic field: THEORY OF TURBULENT DYNAMO (Parker 1955, 1971; Moffat 1978; Krause & Rädler 1980; Ruzmaikin, Shukurov, Sokoloff 1988; review articles by:
  L. Widrow, 2002, Rev. Mod. Phys. 74, 77
  R. Beck , Astron. Astrophys. 24, 4, 2016

The mean-field dynamo:

1. Differential rotation

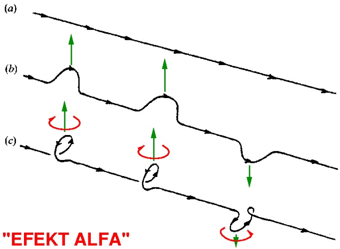
azimuthal stretching of frozen-in m.f. $B_r \rightarrow B_\varphi$



$G = r d\Omega/dr$ - shearing param.

2. Cyclonic turbulence

Convective motions + Coriolis force $\Rightarrow B_\varphi \rightarrow B_r$



$\alpha_t = -\frac{1}{3}\langle \vec{V}' \cdot (\vec{\nabla} \times \vec{V}') \rangle \tau$ - mean fluid helicity

$\eta_t = \frac{1}{3}\langle \vec{V}' \cdot \vec{V}' \rangle \tau$ - turbulent diffusivity

τ - turbulence correlation time

3. Dissipation process necessary: smoothing of magnetic field structure – conversion of loops into the large-scale field $B_{\mathrm{turb}} \sim \langle B \rangle$

Consider a rotating, turbulent, ionised object, (e.g. a star or a disk) pervaded by magnetic field

- Induction equation

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{V} \times \vec{B}\right) + \lambda \nabla^2 B$$

- Ohm's law

$$\vec{j} = \sigma \left(\vec{E} + \frac{\vec{V}}{c} \times \vec{B}\right)$$

- Let us split $\vec{V}$, $\vec{B}$, $\vec{E}$, $\vec{j}$ into the **mean and a fluctuating parts**.

$$\vec{V} = \langle\vec{V}\rangle + \vec{V'} \qquad \vec{B} = \langle\vec{B}\rangle + \vec{B'} \qquad \vec{j} = \langle\vec{j}\rangle + \vec{j'}$$

$\vec{V'}$, $\vec{B'}$ and $\vec{j'}$ represent the **turbulent** velocity, magnetic field and electric current density components.

- Assumption: $\frac{|\vec{B'}|}{|\langle\vec{B}\rangle|} \ll 1$,

- The averaged induction equation

$$\frac{\partial\langle\vec{B}\rangle}{\partial t} = \vec{\nabla} \times \left(\langle\vec{V}\rangle \times \langle\vec{B}\rangle + \vec{\mathcal{E}}\right) + \lambda\nabla^2\langle B\rangle$$

- The averaged Ohm's low

$$\langle\vec{j}\rangle = \frac{1}{\eta}\left(\langle\vec{E}\rangle + \frac{1}{c}\langle\vec{V}\rangle \times \langle\vec{B}\rangle + \frac{1}{c}\vec{\mathcal{E}}\right)$$

$\vec{\mathcal{E}} = \langle\vec{V}' \times \vec{B}'\rangle$ – mean electromotive force of turbulence

Calculation (up to 2nd order in V'): $\vec{\mathcal{E}} = \alpha_t\langle\vec{B}\rangle - \lambda_t\vec{\nabla} \times \langle\vec{B}\rangle$

mean helicity of turbulence: $\alpha_t = -\frac{1}{3}\langle\vec{V}' \cdot (\vec{\nabla} \times \vec{V}')\rangle\tau$

turbulent diffusivity: $\lambda_t = \frac{1}{3}\langle\vec{V}' \cdot \vec{V}'\rangle\tau$

τ - mean correlation time of turbulence

MEAN FILELD DYNAMO EQUATION - Parker (1955)

$$\frac{\partial\langle\vec{B}\rangle}{\partial t} = \vec{\nabla}\times\left(\langle\vec{V}\rangle\times\langle\vec{B}\rangle\right) + \vec{\nabla}\times\left(\alpha_t\langle\vec{B}\rangle\right) + (\lambda_t+\lambda)\nabla^2\langle B\rangle$$

In typical astrophysical conditions $\lambda_t \gg \lambda$
THIN DISK APPROXIMATION ($H/R \leq 1/10$)

$$\frac{\partial\langle B_r\rangle}{\partial t} = -\frac{\partial}{\partial z}(\alpha_t\langle B_\varphi\rangle) + \lambda_t\frac{\partial^2\langle B_r\rangle}{\partial z^2}$$

$$\frac{\partial\langle B_\varphi\rangle}{\partial t} = G\langle B_r\rangle + \lambda_t\frac{\partial^2\langle B_r\rangle}{\partial z^2}$$

THESE EQUATIONS DESCRIBE THE $\alpha\omega$–DYNAMO

$G = r\frac{d\Omega}{dr}$ – rotational shear – generation of B_φ from B_r
α_t – generation of B_r from B_φ
λ_t – turbulent diffusivity – losses of magnetic field from the disk

$$L_t \sim 100\text{pc} \,, \qquad V_t \sim c_s \sim 10\text{km s}^{-1}, \qquad G \simeq \Omega \simeq 10^{-15}\text{s}^{-1}$$

$$\Rightarrow \alpha \sim 1/3 L_t \Omega \sim 1\text{km s}^{-1}, \qquad \eta_t \sim 1/3 V_t L_t \sim 10^{26}\text{cm}^{\,2} s^{-1}$$

(see Ruzmaikin, Shukurov, Sokoloff 1988)
Dynamo number: $D = \alpha_t G H^3/\lambda_t^2$

When $D \geq D_{\rm crit} \sim 10$ the dynamo equation admits exponentially growing solutions:

$$\langle \vec{B} \rangle \propto \exp\left(\frac{t}{t_{\rm dynamo}}\right)$$

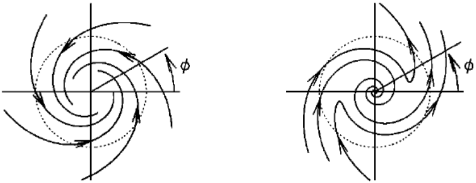
IN GALAXIES:

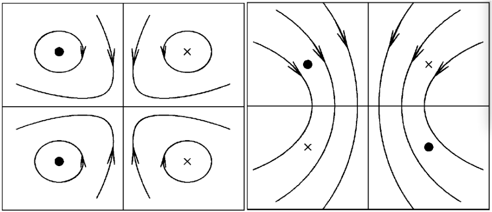
$$t_{\rm dynamo} \sim \frac{H^2}{\lambda_t}\left[\frac{D}{D_{\rm crit}} - 1\right]^{-1/2} \sim 0.5 \div 1 \cdot 10^9 \text{ lat}$$

COMBINED ACTION OF TURBULENCE AND DIFFERENTIAL ROTATION CAN AMPLIFY THE REGULAR MAGNETIC FIELDS

AxiSymmetricSpiral
ASS (m=0)

BiSymmetricSpiral
BSS (m=1)





S0 (quadrupolar)

A0 (dipolar)