# Conceptual foundations and issues in cosmology

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Cosmology = cosmophysics — union of theoretical physics (description of the Universe) and astronomy (observations of distant systems of large objects).
All natural sciences: external observer and object under investigation.
Cosmology: we are a small part of the object ⇒ psychological and epistemic problems — as in social sciences.
As none of other sciences is cosmology based on philosophy and generates philosophical issues.

The core issues are:

1. What constitutes an explanation in cosmology?
2. How do we test validity of our explanations?
3. Are the notions from physics adequate for cosmology?
4. Is the knowledge of the U. as a whole as firm as is the laboratory physics?

General answer to issue 4:
our knowledge of the U. as a whole is ,,softer” than astrophysics and there are *cosmological myths* adequate for our time.

# Cosmology is a mathematized historical science

The U. is observed at large spacetime distances and is measured on and inside the *past light cone* from a tiny part of the Earth's worldline. Experiments cannot be carried out. Fig. 1

Observational cosmology is analogous to:

geology, palaentology, archeology, polotical and economic history — but it has mathematical models ⇒ history of the U. is more precise than in geology and human history.

The history of each country, continent, mountain chain is *unique* — but there are some similarities between histories of different nations, continents and mountains ⇒ general statements in history, geography, biology.

The U. is *unique* — there is nothing to be compared with it.

In physics proper, predictions mean foretelling *future* states from the present state.

In cosmology, predictions of the cosmological future are pointless — cannot be verified. Fig. 2

Cosmology is directed to the past: *prediction* means a conclusion from present observations to *past* times — ,,cosmologists are prophets for the past".

**Core issue no. 3: are the notions taken from physics adequate for cosmology?**

What is the distance to a remote galaxy? What is its velocity w.r.t. us?

# Cosmological distances

The distance between points — well defined in lab. physics. In cosmology: ,,the ladder of cosmological distances" — of what?
Solar system, flat spacetime: radar echo distance to planets. Fig. 3
Curved spacetime, distance to a remote galaxy. Fig. 4
Robertson–Walker spacetime:

$$ds^2 = c^2\,dt^2 - R^2(t)[dr^2 + f_k^2(r)(d\theta^2 + \sin^2\theta\,d\phi^2)],$$

$$f_k(r) = \begin{cases} r & \text{for } k = 0 \\ \sin r & \text{for } k = +1 \\ \sinh r & \text{for } k = -1. \end{cases}$$

Geometry: distance from P to Q is the length of a spacelike geodesic $\gamma_s$ joining P and Q,

$$d_g(P, Q) \equiv \int_{\gamma_s} d\sigma, \qquad d\sigma^2 = -ds^2 > 0.$$

$(x^\alpha) = (ct, r, \theta, \phi)$, $x^\alpha = x^\alpha(\sigma)$, tangent vector $k^\alpha = \frac{dx^\alpha}{d\sigma}$. Spacelike geodesic: the *shortest spacelike* curve from P to Q:

$$\frac{d^2x^\alpha}{d\sigma^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{d\sigma} \frac{dx^\nu}{d\sigma} = 0,$$

$\Gamma^\alpha_{\mu\nu}$ — Christoffel symbols for $g_{\mu\nu}$.
Which curve is the geodesic?

Let P and Q lie on a radial line and $r_P = 0$. Let C: radial curve of simultaneous points,
$x^0 = \mathrm{const} = ct_P$, $r = F(\sigma)$, $\theta = \pi/2$, $\phi = 0$.
C is NOT a geodesic! (Frequent error.)
The *proper distance* is the length of C between P and Q:

$$d_p(P, Q) = R(t_P)\, r_Q.$$

$d_p$ — NOT a geometric distance.
$R(t)$ grows in time $\Rightarrow$ on $\gamma_s : t < t_P$. Fig. 4
Solving the geodesic eq. is hard. Any solution is *useless*:
$d_g(P, Q) = d_g(r_Q)$, $r_Q$ — unknown! (Example: Cartesian grid on the blackboard.)
$d_g$, $d_p$ — NONMEASURABLE!

Two conditions on the physical distance:
— geometrically meaningful (geodesic length),
— measurable in practice.
In cosmology: *cannot both* hold. Commonly accepted concept: introduce a measurable ,,pseudo–distance".
Taken from classical astronomy in flat spacetime — 4 equivalent distances.
In cosmology:
— *luminosity distance* $d_L$,
— *area (angular diameter) distance* $d_A$.

# Luminosity distance

$L$ — absolute luminosity (energy emitted per 1 s) of G, known,
$l$ — apparent luminosity (energy received by the unit area of the telescope), measured.
Space $\mathbf{E}^3$:

$$l = \frac{L}{4\pi d_L^2}.$$

Def. In R–W cosmology

$$d_L \equiv \left(\frac{L}{4\pi l}\right)^{1/2}.$$

$t_1$ — emission of light by G, $t_0$ — observation of G on Earth, $t_0 > t_1$,
$r_G$ — radial coord. of G, $r = 0$ — on Earth. In R–W geometry

$$d_L = \frac{R^2(t_0)}{R(t_1)}\, f_k(r_G).$$

# Area distance

As in $\mathbf{E}^3$. In R–W

$$d_A = R(t_1)\, f_k(r_G).$$

Test of isotropy and homogeneity of space (of R–W geometry):

$$\frac{d_L}{d_A} = (z+1)^2.$$

Farthest galaxy observed: $z \cong 7 \Rightarrow d_L \cong 64\, d_A$.
$d_L$, $d_A$ — *qualitative* indicators of distance.
$d_L$ is better: if $R$ grows monotonically $\Rightarrow d_L$ grows monotonically with increase of $r_G$; $d_A$ — may have maximum for growing $r_G$ and decreasing $R(t_1)$.

# What is velocity of a galaxy w.r.t. the Earth?

Unclear how to define.
1. The change of geodesic distance: $v_g = \frac{d}{dt} d_g$ — nonmeasurable.
2. Change of the proper distance: $v_p = \frac{d}{dt} d_p = \frac{d}{dt}(Rr) = H(t) d_p$ — nonmeasurable.
For nearby galaxies $d_p \cong d_L$, gravit. redshift $\equiv$ Doppler redshift $\Rightarrow v/c = z \Rightarrow$ *linear* Lemaître–Hubble law

$$v \cong cz = H_0 d_L.$$

# What is the Universe?

Principal issue: *there is only one physical U.*
String/M theory ⇒ *multiverse* — metaphysics in mathematical disguise.
Various definitions.
1. ,,The largest set of objects and events to which physical laws can be applied consistently and successfully." (H. Bondi 1960)
2. ,,In cosmology we try to investigate *the world as a whole* and not to restrict our interest to closed subsystems (Earth, Solar system, the Galaxy)." (Sexl and Urbantke 1983)
3. ,,The U. is all that there is." (G. Ellis 1014)
What is ,,all"? For whom? Example in SR: inertial observer and uniformly accelerated observer (hyperbolic motion). Fig. 5
What exists (,,there is")? Objects under the event horizon of a BH?
Set theory: Cantor's antinomy — ,,set of all sets" — contradictory notion.
Are these definitions logically consistent?

Math. def.:
,,The U. is the maximal analytic extension of some exact solution of Einstein field eqs. belonging to the class of cosmological solutions".
(implicitly Hawking and Ellis 1973)
Consequences.
a) there are domains of the spacetime which are fundamentally inaccessible to our observations: under the horizons of BH, the other exterior domains (Kerr, Reissner–Nordström).
b) The solutions are structurally unstable if $J \to 0$ and/or $Q \to 0 \Rightarrow$ analyticity is unrealistic.
c)The def. is based on presently accepted physical theory (GR).

General remark:
in cosmology we *cannot* use and we *never* use exact solutions.
All the definitions are incomplete and vague.
Conclusion:
the U. is *not* a well defined physical object.
Most cosmologists do not worry about it and in *everyday practice* cosmology goes well without it.
The world as a whole is not accessible empirically.
Practical concept: ,,*observable U.* $=$ *metagalaxy*". (H. Alfven 1967)

# Uniqueness of the Universe

Only one U. is described (in principle) by $\infty$ of mathem. models in GR $\Rightarrow$ the theory is over–excessive w.r.t. the world.
Adequate: 1 world — 1 theory with only one solution (state) — that observed.
Impossible. All sciences: unlimited number of objects $\Rightarrow$ scientific theory with distinction:
general laws with $\infty$ number of possible states $\leftrightarrow$ initial (boundary) conditions determined by observations establish which state occurs in reality.
Laws of physics — necessity,
initial conditions — contingent (chance).
One U. $\Rightarrow$ no distinction between laws and initial conditions.

# 4 consequences of the uniqueness of the U.

(G. Ellis 2007)

**Thesis 1**

*The U. itself cannot be subjected to physical experimentation*.

We can only passively observe it along our past light cone.

We cannot re–run the U. with altered initial conditions ⇒ the actual initial conditions for it are *absolute* and *unchangeable* — but we believe (?) they are contingent.

**Thesis 2**

*The U. cannot be observationally compared with other universes*.
There is NO statistical ensemble of physically existing other universes $\Rightarrow$ we cannot establish if actual evolution of the U. is ,,typical" — e.g. the structure formation.

**Thesis 3**

*The concept of ,,Laws of Physics" that only apply to the U. as a whole is questionable*

There is no discrimination: laws $\leftrightarrow$ initial conditions $\Rightarrow$ there are no ,,laws determining the initial conditions for the U.".

Genuine physical laws apply locally to the constituents of the U. — atoms, stars, galaxies.

**Thesis 4**

*The concept of probability cannot be applied to the unique U.*

Observationally we cannot check if our U. is „typical" — we see no other worlds. Theoretically we cannot say whether our U. is less or more probable.

Common opinion up to $\approx$ 1980:

compare the U. with the math. statistical ensemble of universes (cosmological solutions to Einstein eqs.). „Typical" universe $\Leftrightarrow$ a generic cosmol. solution.

R–W geometry — the most specialized (highest symmetry) solution $\Rightarrow$ extremely exceptional. Why is our U. so little probable?

Now:

comparing the real U. with the mathem. ensemble makes no sense $\Rightarrow$ do not ask if the U. is typical or not. Instead: physics *should* explain why the U. is as it is — at present beyond our reach.

Conclusion:
the philosophical idea: the U. is *generic* (*typical, probable*), or the opposite, cannot be formulated in physics and is unverifiable.

FINE TUNING

Our U. is exceptional also in the class of R–W spacetimes: is very large, very flat and very old ($\Leftarrow$ inflation) and there are **fine tunings**.
If values of two independent phys. quantities are fine tuned — this requires some explanation $\Leftarrow$ it is less probable that it is by chance.
,,Probable or not" — this requires a statistical ensemble of objects.

In cosmology:

— $\rho_{\rm DE} \cong (2-3)\rho_{\rm DM}$,

— the U. admits existence of life (is ,,biophilic").

Standard explanation based on Weak Anthropic Principle:

a) there is a statistical ensemble of universes with different values of $\rho_{\rm DE}$, $\rho_{\rm DM}$ and nuclear constants,

b) we could only appear in a universe which admits us, though this world is less probable,

(a common sense argument).

The ensemble does NOT exist $\Rightarrow$ in the unique U. no object or event is much or less probable. The cosmol. fine tunings need a different explanation.

None physical law excludes fine tunings.

# Nuclear forces, the U. and life

Physical parametes determining the strength of nuclear forces (strong interactions) are crucial for the structure of the U. and life.
Fundamental:
— helium has only 2 stable isotopes, $^{3}\mathrm{He}$ and $^{4}\mathrm{He}$,
— isotopes $^{2}\mathrm{He}$ („diproton") and $^{5}\mathrm{He}$ are extremely short living,
— there are no long–living nuclei with $A = 5$ and 8.
Were the nuclear forces stronger $\Rightarrow$ in primordial nucleosynthesis all H would be burnt into He and all heavier elements $\Rightarrow$ today: only dust and small solids, no sources of energy, no stars, only cold ($T \approx 3$ K) and darkness $\Rightarrow$ no life.
Sufficient for this change: only small changes of physical constants.

# Model building for the Universe

Apart from these problems: cosmology is a physical science $\Rightarrow$ models of the U. can be falsified — the steady–state cosmology. Like in historical science: falsification means that the interpretation of the history of the U. has been wrong and must (and can) be revised.

Observations $\Rightarrow$ **Copernican (Cosmological) Principle** (H. Bondi 1960): *our position in space is not specially distinguished in any way*.

Direct observations: on large scales the U. is isotropic around us.

Indirect evidence: the U. is approximately isotropic about every point $\Rightarrow$ homogeneity of space.

Mathematical form of CP:

*the U. is spherically symmetric and spatially homogeneous* $\Rightarrow$ R–W spacetime.

Space is isotropic and homogeneous for the family of *fundamental observers* — comoving with the cosmic matter (centers of large galaxy clusters).

Fundam. observers are at rest in the space: $r, \theta, \phi =$const, their worldlines are coordinate lines of time $t$.

Today: the rest frame of the fundam. observers is identical with the rest frame of the CMB relic radiation.

Any observer moving w.r.t. the fundamental ones sees the space anisotropic and inhomogeneous: Doppler shift of stellar light, temperature of the relic radiation is $T = T(\theta)$.

R–W spacetime *evolves*: the space expands or contracts in time.

R–W geometry: 3 spacetimes with different spaces,

$$ds^2 = c^2\, dt^2 - R^2(t)[dr^2 + f_k^2(r)(d\theta^2 + \sin^2\theta\, d\phi^2)],$$

$k = 0$ — euclidean space $\mathbf{E}^3$,
$k = +1$ — 3–sphere (closed space),
$k = -1$ — Lobatchevski space (open),
all simply connected. Possible other topologies — seldom used in cosmology.
$R(t)$ — *cosmic scale factor*, any smooth function.
$[r] = 1$, $[R] =$length.

# Dynamics

Einstein field eqs. (EFE):

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

$T_{\mu\nu}$ — any matter (inc. $\Lambda$).
Geometry of R–W $\Rightarrow$ $T_{\mu\nu}$ has symmetries of the metric $\Rightarrow$

$$T_{\mu\nu} = (c^2\rho + p)u_\mu u_\nu - p g_{\mu\nu}$$

*perfect fluid*, $c^2\rho$ — energy density, $p$ — pressure, $c \equiv 1$,
$u^\mu = (1000)$ — 4–velocity of comoving droplets of cosmic fluid (of fundam. observers).
Usually: $p = p(\rho)$ — *barotropic* EOS.
Vacuum state of a quantum field: $T_{\mu\nu}(vac) = \rho_v\, g_{\mu\nu}$,
$\rho_v = -p_v = \mathrm{const} > 0 \Leftarrow \nabla_\nu T^{\mu\nu} = 0$ — relat. hydrodynamics.

Cosmol. constant:

$$\Lambda \mapsto T_{\mu\nu}(\Lambda) = \rho_\Lambda \, g_{\mu\nu}, \qquad \rho_\Lambda \equiv \frac{\Lambda}{8\pi G}$$

$\Rightarrow \Lambda \Leftrightarrow$ vacuum state in QFT.
Classical matter (not quantum vacuum) in GR is *arbitrary* — it should satisfy *energy conditions*: WEC, SEC, DEC.
Physically very reasonable conditions — hold for all known matter.
SEC is broken in a prefect fluid: if $\rho = 1g/cm^3$ and $p < -10^{15}$ atm.

# Global evolution

GR: energy conditions determine *global evolution* of the spacetime.
GR: *generic singularity theorems* (Hawking and Penrose $\approx$ 1970):
*under reasonable phys. conditions singularities develop in the spacetime*.
**What is singularity?**
No complete and fully satisfactory definition. Singularity theorems are expressed in terms of geodesic incompleteness. In a spacetime free of singularities each timelike and null geodesic may be infinitely extended to the future and past. If a timelike or null geodesic is stopped at a point and cannot be extended farther — it met a singularity.
Spacetime has an ,,edge" — particle or a photon history (worldline) has a beginning or an end.
Singularity — NOT a point: *a singular boundary of the spacetime* $\Rightarrow$ a singular 3–dim. hypersurface — a 3–dim. set of points.
Example: initial singularity in R–W geometry:
the set of points $(r, \theta, \phi)$ at $t = 0$ — hypersurface *metrically* contracted to a point $\Rightarrow$ all distances are $= 0$.

Schwarzschild singularity ($r = 0$), initial and final ($k = +1$) singularities in R–W are *spacelike*.
Spacelike hypersurface: the orthogonal vector $n^\alpha$ is *timelike*, $n^\alpha n_\alpha > 0$.
Singularities are NOT due to high symmetry — they are common in physically reasonable spacetimes. Regular spacetimes (free of singularities) are exceptional — plane gravit. waves.
Singularities (geodesic incompleteness) are frequently also curvature singularities:
$R_{\alpha\beta\mu\nu}$ — 14 scalar algebraic invariants: $R^\mu{}_\mu$, $R_{\alpha\beta}R^{\alpha\beta}$, $R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$, ... .
*Curvature singularity*: some of the scalars diverge to $\infty$. Schwarzschild BH:

$$R_{\alpha\beta} = 0, \qquad R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = \frac{48M^2}{r^6} \to \infty \quad \text{for} \quad r \to 0.$$

For known cosmological spacetimes singularity is a curvature sing.

# Singularities in R–W geometry

Matter = perfect fluid.
WEC: $\rho \geq 0$,
SEC: $\rho + p \geq 0$ and $\rho + 3p \geq 0$.
Singularity theorem:
if $\Lambda \leq 0$, SEC holds and $H(t) = \dot{R}/R > 0$, then a curvature singularity occurs in a finite past of any point.
Very early U.: matter ultrarelativistic $\Rightarrow \rho = 3p$ and
$R^{\mu}{}_{\mu} = -8\pi G(\rho - 3p) = 0$,
yet $R_{\alpha\beta}R^{\alpha\beta} = (8\pi G)^2(\rho^2 + 3p^2)$ — when all the propagation eqs. are used.
If $R(t) \to 0$ for $t \to 0 \Rightarrow$ all volumes are
$\propto R^3 \to 0 \Rightarrow \rho \to +\infty \Rightarrow R_{\alpha\beta}R^{\alpha\beta} \to +\infty$ — curvature singularity.
Also $|p| \to \infty$ uniformly in the space. Large $p$ *cannot* prevent the collapse $R \to 0$ for $t \to 0$ — NO pressure gradient.

Is it possible to avoid the singularity in R–W geometry?
3 cases.
1. $\Lambda > 0$. The relic CMB radiation has $z \approx 1000 \Rightarrow$ the expansion from the recombination epoch to now is $R(t_0)/R(t_{rec}) \approx 1000$. It may be shown that the singularity can be prevented if

$$\rho_\Lambda \equiv \frac{\Lambda}{8\pi G} > (1000)^3 \, \rho_M(t_0)$$

observationally excluded.
2. SEC broken: $\rho + 3p < 0$ — requires extreme quantum effects, unlikely.
3. A different theory of gravity. Now fashionable, but all data $\Rightarrow$ GR is the best.

# Initial singularity in R–W by elementary calculations

EFE $\Rightarrow$ ordinary eqs. for $R(t)$ and fluid variables. $\Lambda = 0$.
Friedmann eq.

$$\dot{R}^2 + k = \frac{8\pi G}{3}\rho R^2,$$

deceleration eq.

$$\ddot{R} = -\frac{4\pi G}{3}(\rho + 3p)R < 0$$

from SEC, eq. motion of the fluid,

$$\dot{\rho} = -\frac{3\dot{R}}{R}(\rho + p),$$

$\dot{\rho} < 0$ if $\dot{R} > 0$ and SEC holds.
Friedmann and deceleration eqs. $\Rightarrow$ NO static solutions, $\dot{R} \neq 0$, U. must evolve.
Observations: $\dot{R}(t_0) > 0 \Rightarrow R$ is growing about $t_0$.

Assume: $\dot{R} > 0$ for $t_m < t \leq t_0$ and $\dot{R} < 0$ for $t < t_m \Leftrightarrow R$ has minimum at $t = t_m$. At minimum:

$$0 < \ddot{R}(t_m) = -\frac{4\pi G}{3}(\rho + 3p)R\Big|_{t_m} \qquad \Rightarrow \; \rho + 3p < 0,$$

— SEC is broken. Must be $\dot{R}(t) > 0$ for $t \leq t_0$ and $\ddot{R}(t) < 0 \Rightarrow$ for $t = \tau$ in a finite past there is $R(\tau) = 0$.
$\tau \equiv 0$ — the *origin* of time, $R(0) = 0$.
$R(0) = 0 \Rightarrow$ all spatial distances between different points are 0,

$$d\sigma^2 \equiv -ds^2\Big|_{t=0} = R^2(0)[dr^2 + f_k^2(r)d\Omega^2] = 0,$$

the space at $t = 0$ is contracted *metrically* to 1 point and for $t > 0$ it *expands* $\Rightarrow t = 0$ is the origin of space $\Rightarrow$ *the origin of the spacetime*.
Initial singularity = Big Bang (Fred Hoyle $\approx$ 1948).

Curvature singularity $\Rightarrow$ geodesic incompleteness. Time and space cannot be extended to $t < 0$ — there is no time, no space, nothing. Fig. 6
Returning to singularity: $t \to 0^+$, all volumes $\to 0$, $\rho \to \infty$, $p \to \infty$, matter is *adiabatically* squeezed and gets *relativistic* $\Rightarrow T \to +\infty$, $p \approx \rho/3$, plasma of element. particles — like photon gas with Planck spectrum.
Soon after BB: extreme $\rho$, $T$, particle energies — physics beyond the Standard Particle Model.
For lower $\rho$ and particle energies: the Standard Model holds, plasma is in *thermal equilibrium* — the simplest physical system — the Early Universe (EU).
EU — much simpler than the U. now.
EU — from $t \cong 10^{-12}$ s to recombination epoch, $t_{rec} \cong 5 \cdot 10^5$ years, $T_{rec} \cong (6000 - 3000)$ K.

For $t < 10^{-12}$ s — *Very Early Universe* (VEU) — very short, it determined most of main features of the present U.
VEU: extreme particle energies and densities, inaccessible in present and future accelerators $\Rightarrow$ physics experimentally untestable.
Cosmology requires physical laws beyond experimental (and observational) reach.
**Physics horizon** (G. Ellis, R. Maartens, M. MacCallum 2012):
*a border that separates the experimentally established and verified physics from physical concepts which will never be tested, especially in extremely high energy processes*.
The physics horizon limits our knowledge of physical laws governing the VEU.

# Speculations on Very Early U. and their limitations

Conjecture: the first era after BB is Quantum Gravity Era. Why QG era? 2 motivations.

1. **Planck units**.

In dim=3+1: all dimensional quantities are built of units of mass M, length L and time T, units L, M, T — arbitrary.

A more physical system of units is based on the 3 fundam. constants of the nature:

$\hbar$ — all matter is quantum,

$c$ — all matter is relativistic,

$G$ — all matter gravitates.

Each phys. quantity Q may be expressed dimensionally as a product

$$[Q] = [\hbar^x c^y G^z] = (\mathrm{g\,cm^2\,s^{-1}})^x (\mathrm{cm\,s^{-1}})^y (\mathrm{cm^3 g^{-1} s^{-2}})^z.$$

Unit of length:

$$\mathrm{cm} = [\hbar^x c^y G^z] \quad \Rightarrow \quad x = z = \frac{1}{2}, y = -\frac{3}{2}.$$

**Planck units** (Max Planck 1900):

$$
\begin{aligned}
l_P &= \sqrt{\frac{\hbar G}{c^3}} = 1,6 \cdot 10^{-33}\mathrm{cm}, \\
t_P &= \frac{l_P}{c} = \sqrt{\frac{\hbar G}{c^5}} = 5,4 \cdot 10^{-44}\mathrm{s}, \\
m_P &= \sqrt{\frac{\hbar c}{G}} = 2,2 \cdot 10^{-5}\mathrm{g}, \\
E_P &= m_P c^2 = 1,2 \cdot 10^{19}\mathrm{GeV}, \\
T_P &= \frac{E_P}{k_B} = 1,4 \cdot 10^{32}\mathrm{K}, \ldots .
\end{aligned}
$$

This scale is either *very large* or *very small* — does not fit known fundam. processes.

$\hbar$, $c$, $G$ — together should be relevant for *quantum gravity* effects.

Conjecture: the Planck units determine the scale at which quantum effects dominate in gravit. interactions.

2. All matter is quantum and gravitation is the only universal interaction $\Rightarrow$ gravit. field should have quantum nature. All observed gravit. fields are weak and we see no quantum effects in them $\Rightarrow$ quantum gravity appears in extremely strong and variable fields $\Rightarrow$ BH singularities and BB singularity.
If quantum gravity effects dominate in the first era after BB $\Rightarrow$ we *need* a theory of quantum gravity.
Einstein 1918, Mark Bronstein ca. 1930, Dirac from ca. 1950, Feynman ca. 1965, many other eminent physicists... — outcome *very modest*.
Nothing but difficulties and obstacles of all possible kinds, no hints from experiment or observations.
To formulate a quantum theory of gravity is definitely the most ambitious and difficult intellectual task in the whole history of mankind.
Conclusion:
about quantum gravity era we know nothing at present and we expect that if the theory is created in future, we will never test it.

# The next era: Grand Unification Era

Classical spacetime exists and gravitation is described by GR. Particle energies and interactions beyond the Standard Model, some Grand Unified theories hidden under the physics horizon, probably never reliably verifiable. Almost nothing is known.
Conjecture: in this era 2 processes occurred — inflation and baryogenesis. Inflation was first.

# Baryogenesis

Why does the U. consist almost solely of matter?
Solar system — only matter.
Cosmic rays: 1 $\bar{p}$ per $10^4$ protons.
We see no particles produced in annihilation process $p + \bar{p} \Rightarrow$ there are NO regions where matter and antimatter collide on macroscopic scale.
Early U.: dense homogeneous plasma of $p$ and $\bar{p}$. We know of no processes segregating $e^- p$ from $e^+ \bar{p} \Rightarrow$ at present there are no anti–stars and anti–galaxies.
Fundam. particle physics: *exact symmetry matter — antimatter*.
VEU after inflation: if it were exactly symmetric $\Rightarrow$ all pairs $e^- e^+$ and $p\bar{p}$ later annihilated into photons and today there would be only the 3 K relic radiation $\Rightarrow$ at $t \approx 10^{-12}$ s there was an excess of matter over antimatter.
How large? Parameter

$$\eta \equiv \frac{n_b - n_{\bar{B}}}{n_\gamma} \cong \mathrm{const}$$

because all number densities evolve as $R^{-3}$.

Primordial nucleosynthesis: abundances od D, $^3\mathrm{He}$ and $^7\mathrm{Li} \Rightarrow \eta \cong 6 \cdot 10^{-10}$.
Relativistic plasma in early U.: 1 unpaired $e^-$ per $10^9$ pairs $e^-e^+$ and 1 unpaired $p$ per $10^9$ pairs $p\bar{p}$. All pairs annihilated in early U., only the excess particles have survived and form the present U.
What did generate the excess after the inflation?
Various concepts.
Some concepts: baryogenesis in Grand Unification Era $\Rightarrow$ hidden behind the physics horizon and untestable.
Other mechanisms: baryogenesis at the beginning of the early U. $\Rightarrow$ in principle is verifiable.

# Horizons and inflation

Historically: inflationary evolution introduced to solve problems with *cosmological horizons*. (Most textbooks careless on them.)

**Visual horizon**

We can see no electromagn. signals emitted before the recombination epoch $\Leftarrow$ photons coupled to the plasma.

In principle we can see primordial neutrinos (beginning of the radiation era) and primordial gravit. waves (earlier) — both in very far future.

# Particle horizon

Arises because there is (in R–W and others) the spacelike initial singularity.
R–W: all particles = galaxies = fundam. observers on timelike geodesics $x^0 = t$, $r, \theta, \phi =$const, E — Earth with $r = 0$. Fig. 7
$H_p = H_p(E, t_0)$ — depends on line E and point $E_0(t_0)$.
$E_0$ can see only events on and inside the past light cone $AE_0B$.
Actually we see only closer particles emitting at $t > t_{rec}$ — visual horizon.
CD — sphere of points on $H_p$ simultaneous with $E_0(t_0)$.
$r_H(t_0)$ — radial coordinate of all points on $H_p(E, t_0)$,

$$r_H(t_0) = c \int_0^{t_0} \frac{dt}{R(t)},$$

converges for Friedmann models.

Radius of $H_p(t_0)$: *proper distance $d_p$ from $E_0$ to points C, D of the simultaneous sphere* — ,,radius" of the sphere CD — NOT the geodesic distance (errors!).

$$d_H(t_0) = R(t_0)\, r_H(t_0) = cR(t_0) \int_0^{t_0} \frac{dt}{R(t)}.$$

Let $k = 0$, $p = w\rho$, $w =$const $\Rightarrow$

$$R = \mathrm{const} \cdot t^{\frac{2}{3(1+w)}}, \qquad d_H(t) = \frac{3(1+w)}{1+3w}\, ct.$$

For $t_0 \approx 13,7 \cdot 10^9$ years and $w = 0$: $d_H(t_0) \approx 4 \cdot 10^{10}\mathrm{ly} \approx 10\mathrm{Gpc}.$
$H_p$ expands faster than light: its radial velocity

$$v_r \equiv \frac{d}{dt}\, d_H(t) = c[1 + H(t) d_H(t)] = \frac{3(1+w)}{1+3w}\, c > c.$$

This not a physical motion. A particle inside $H_p(t_0)$ cannot escape outside it for $t > t_0$.

We cannot see what occurs outside $H_p$, but we feel the gravit. field of all matter outside $H_p$.

# Cosmological event horizons

Also in geometries other than R–W.
$\mathcal{J}^-$ — spacelike initial singularity,
E — timelike worldline emanating from $\mathcal{J}^-$ (not necessarily a geodesic). Fig. 8
**Cosmol. past event horizon** $H^-(E)$ — *future light cone* of the initial point of E.
Inside $H^-$ — events may be affected and seen by E,
outside $H^-$ — events cannot be affected by E (can be seen).
$\mathcal{J}^+$ — spacelike future (final) singularity,
E — worldline ends at $\mathcal{J}^+$. Fig. 9
**Cosmol. future event horizon** $H^+(E)$ — *past light cone* of the final point of E.
Outside $H^+$ — events invisible for E.
In some cosmological spacetimes both the event horizons can occur: R–W for $k = +1$.

# Problems of the decelerating early U.

SEC holds $\Rightarrow \ddot{R} < 0$ in the past $\Rightarrow R \propto t^q$, $0 < q < 1$. Slow expansion $\Rightarrow$ the horizon problem.
Furthermore: Friedmann spacetimes are very special even in the class of R–W geometries: why is our U. so flat, large and old?
These problems may — in principle — be solved by an inflationary evolution $\Leftarrow$ they are related to the horizon problem.

# The horizon problem

Does not exist in pure R–W geometry — it appears if R–W arises by smoothing out a generic spacetime.

We observe 2 antipodal points (poles) on the the celestial sphere at $t = t_{rec}$ — this is the *last scattering surface* (LSS) for us at $t = t_0$. Is the radius of LSS larger or smaller than $d_H(t_{rec})$ of $H_p(t_{rec})$? Fig. 10

Null geodesics $\Rightarrow r_C = r_D = r_H(t_{rec})$.

The diameter (the proper distance) of $H_p(t_{rec})$ is $l_{CD} = 2d_H(t_{rec})$, for ultrarel. matter $w = 1/3 \Rightarrow l_{CD} = 2 \cdot 2ct$.

The diameter (proper distance) of LSS is $l_{NS} = 2r_N R(t_{rec})$.

$r_N = r_S$ — determined by the incoming radial null geodesic $NE_0$,

$$r_N = \int_{r_N}^{0} c\frac{dt(r)}{R} = c\int_{t_{rec}}^{t_0} \frac{dt}{R}.$$

In galactic era: $p = 0 \Rightarrow R = at^{2/3}$ for $k = 0 \Rightarrow$

$$r_N = \frac{3c}{a}(t_0^{1/3} - t_{rec}^{1/3}) \Rightarrow l_{NS} = 6ct_{rec}\left[\left(\frac{t_0}{t_{rec}}\right)^{1/3} - 1\right].$$

The ratio

$$\frac{l_{NS}}{l_{CD}} = \frac{3}{2}\left[\left(\frac{t_0}{t_{rec}}\right)^{1/3} - 1\right].$$

For $t_0 \approx 13,7 \cdot 10^9$ y, $t_{rec} \approx 5 \cdot 10^5$ y: $l_{NS}/l_{CD} \approx 45$.
LSS was covered with disks arising by intersections of LSS with spheres of local particle horizons at $t = t_{rec}$. Fig. 11
The number of the disks is

$$\approx \frac{4\pi(\frac{1}{2}l_{NS})^2}{\pi d_H^2} = \left(\frac{l_{NS}}{d_H}\right)^2 = 4(45)^2 \approx 8000.$$

If signals with velocity $\leq c$ were emitted from a point Q at $t = 0$, all points in the disk with the centre at Q are causally connected at $t = t_{rec}$.
Interiors of different disks cannot interact up to times $> t_{rec}$.
Let the U. have R–W geometry since $\approx 10^{-42}$ s — beginning of GU Era.
Then the outcome $l_{NS}/l_{CD} \gg 1$ is innocuous.

The problem is if:
Conjecture (early 1970's): in GU Era the spacetime was irregular (a ,,generic" solution of EFE) and at this time a smoothing process started at some point Q. Up to $t_{rec}$ only one disk on LSS would be smoothed out to R–W geometry ⇒ entire LSS would be highly irregular — excluded by the isotropy of the relic CMB radiation.
This is the *horizon problem*.
Conclusion:
the R–W geometry of the U. cannot be the outcome of a local smoothing process and this geometry must have been in the whole space soon after BB.
How soon? It is model dependent.

# Inflation solves the horizon problem

Introduce an *inflationary epoch*: a short period in VEU of very fast (exponential) growth of $R(t)$ to make the radius $d_H(t_{rec})$ of $H_p$ larger than the radius of LSS. Fig. 12

Assumptions ($k = 0$):

a) for $0 < t \leq t_1$ matter is ultrarelat. $\Rightarrow R = at^{1/2}$,

b) for $t_1 < t < t_2$ the scale factor $R$ grows $Z$ times, $R(t_2) = ZR(t_1)$, $Z > 10^8$,

c) for $t_2 < t < t_{rec}$ matter is ultrarelat. $\Rightarrow R = bt^{1/2}$,

d) for $t > t_{rec}$ matter is nonrelat. dust $\Rightarrow R = \gamma t^{2/3}$.

$$\text{Then} \quad r_H(t_{rec}) = c \int_0^{t_1} \frac{dt}{R} + c \int_{t_1}^{t_2} \frac{dt}{R} + c \int_{t_2}^{t_{rec}} \frac{dt}{R}.$$

$\Delta t = t_2 - t_1$ -very short $\Rightarrow \int_{t_1}^{t_2}$ is neglected,

$$l_{CD} = 2d_H(t_{rec}) = 4ct_{rec} \left[ (Z-1) \left( \frac{t_1}{t_{rec}} \right)^{1/2} + 1 \right].$$

If

$$Z \geq \frac{3}{2}(t_0^2 t_{rec} t_1^{-3})^{1/6} \quad \Rightarrow \quad \frac{l_{NS}}{l_{CD}} \leq 1.$$

$t_1$ — free parameter (model dependent):
$t_1 = 1s \Rightarrow Z > 1,5 \cdot 10^8$,
$t_1 = 10^{-39}s \Rightarrow Z > 5 \cdot 10^{28}$.
Inflationary evolution from $t_1$ to $t_2$:

$$R = Ae^{H(t-t_1)}, \quad A = \mathrm{const}, \quad H = \mathrm{const},$$

$$\Delta t = t_2 - t_1 \quad \Rightarrow \quad H = \frac{\ln Z}{\Delta t}.$$

One of the models: $t_1 \cong 10^{-36}$ s, $t_2 \cong 10^{-32}$ s $\Rightarrow Z > 10^{26}$ and $H \cong 2 \cdot 10^{53} \mathrm{km\,s^{-1}Mpc^{-1}}$.