Star Formation: Now and Then

Denis Burgarella, Astronomer

Laboratoire d'Astrophysique de Marseille, France

denis.burgarella@lam.fr

SECOND COSMOLOGY SCHOOL

INTRODUCTION TO COSMOLOGY
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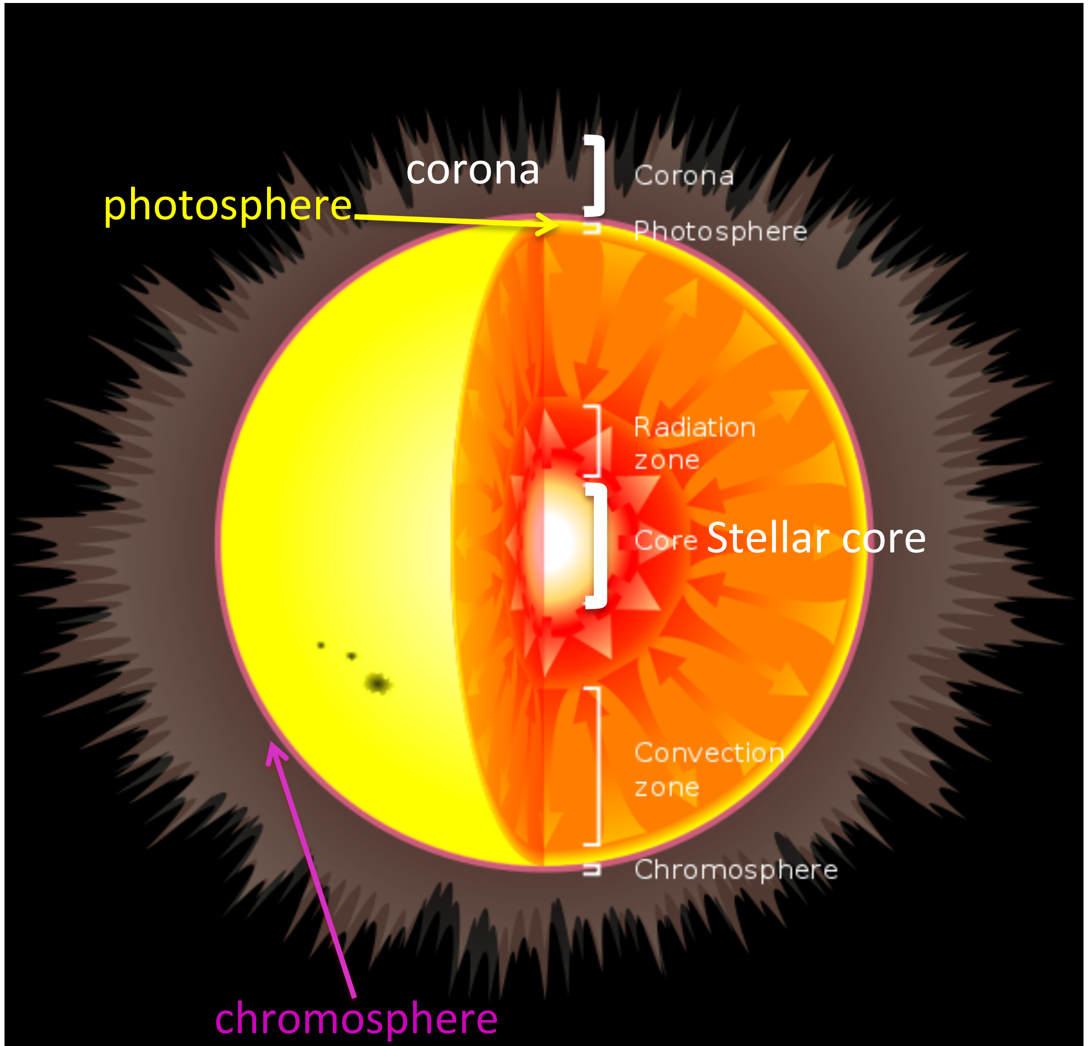
Overview

What kind of information is relevant for the first population of stars appearing in the universe?

- The Initial Mass Function
- Cooling and coolants
- The Jeans instability
- Fragmentation of the clouds
- Detection

The building blocks in the universe: stars



Star clusters

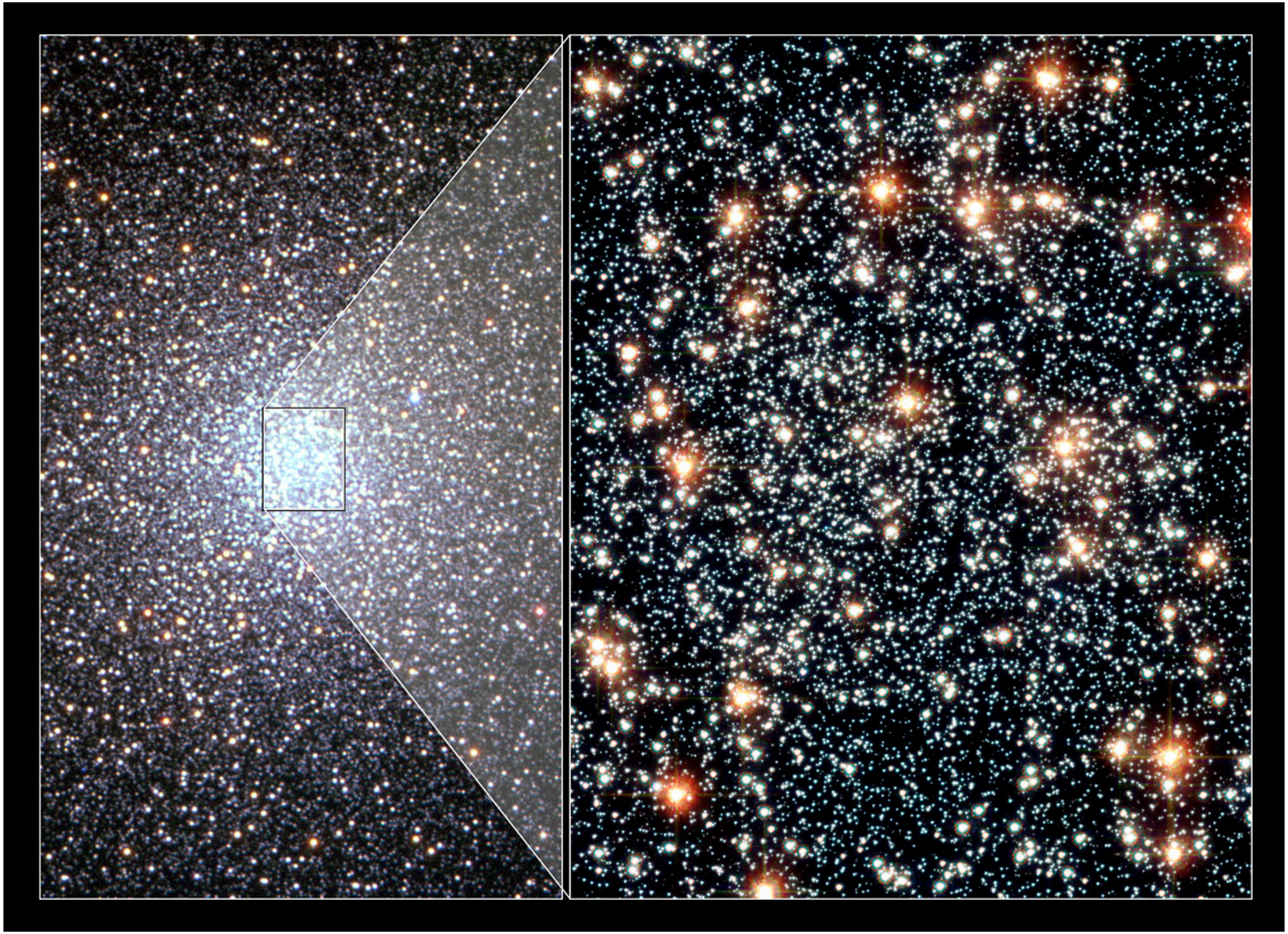
NGC3603 from Hubble Space Telescope

Star clusters are very useful to understand stars and their evolution:

- Stars all at same distance
- Stars are dynamically bound
- Stars have same age
- Stars have same chemical composition

Can contain $10^3 – 10^6$ stars

Globular clusters are more massive star clusters



Observable properties of stars

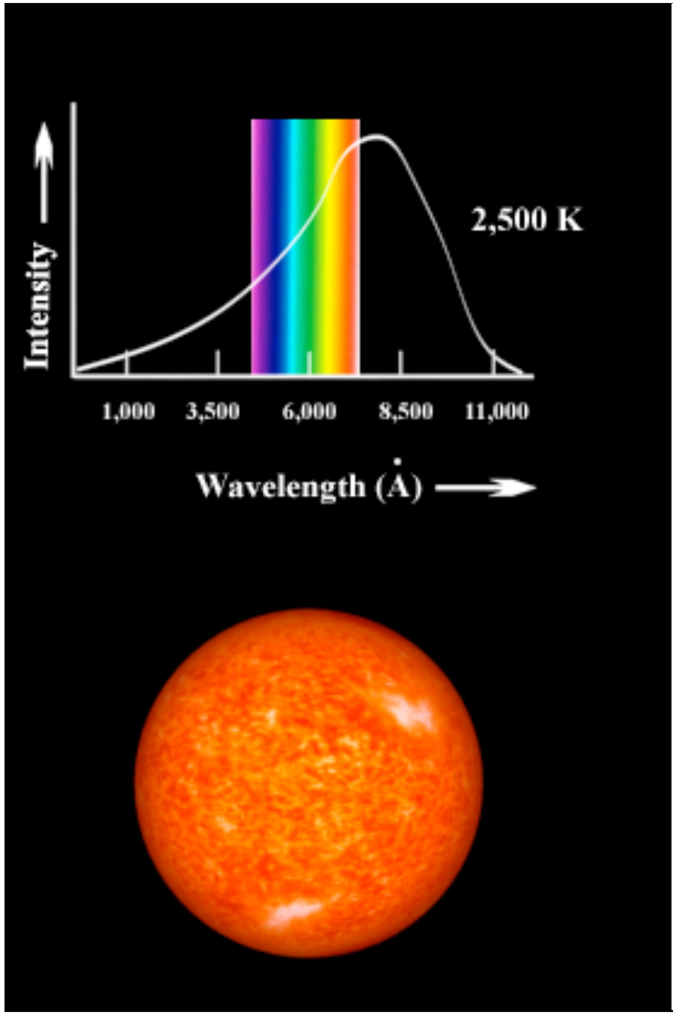
Basic parameters to compare theory and observations:

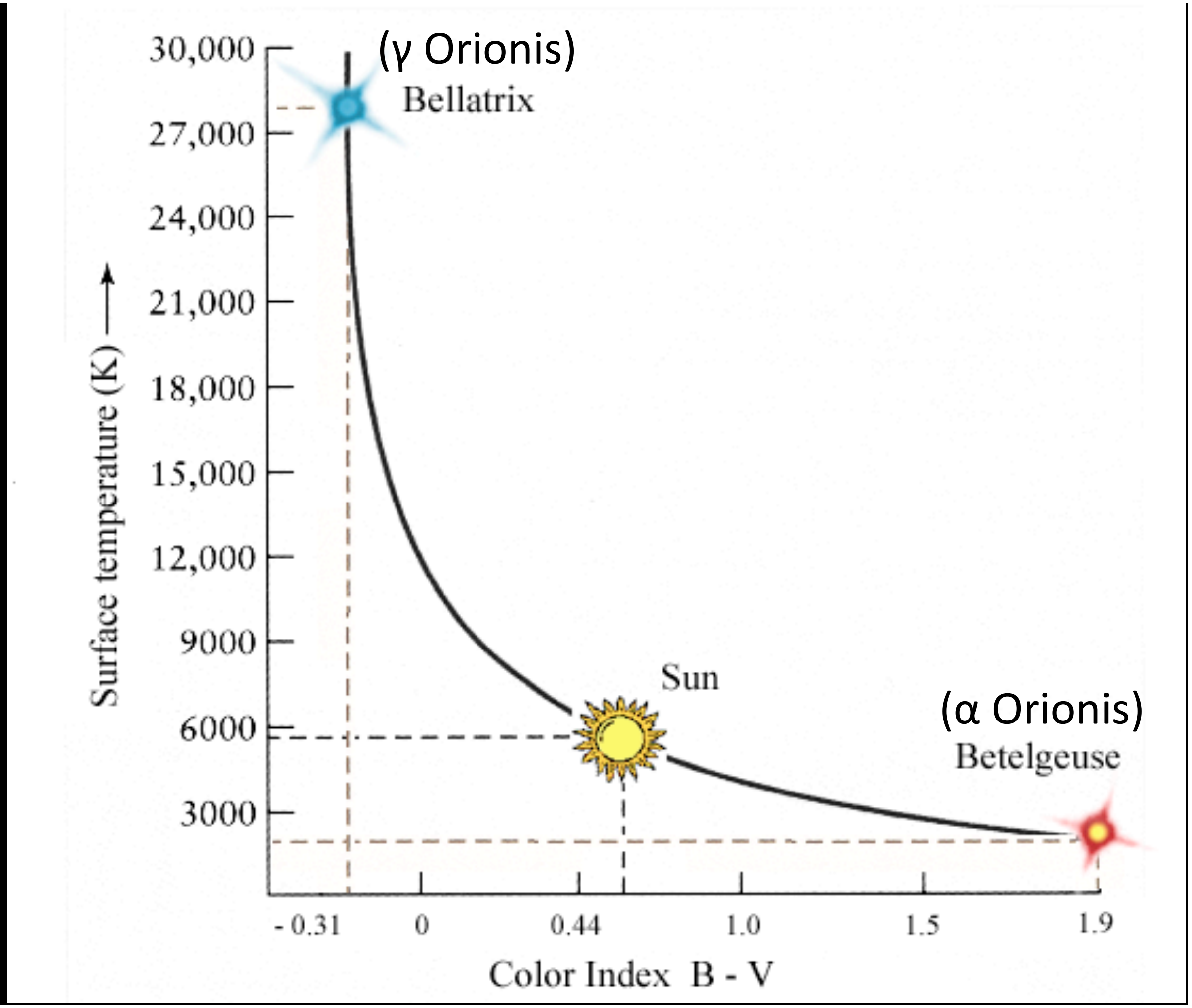
- Stellar mass (*M*)
- Luminosity (*L*)
  - The total energy radiated per second i.e. power (in W): $L = \int_0^\infty L_\lambda d\lambda$
  - If *F* is the flux ($W \cdot m^{-2}$) and *L* is the luminosity (W) or where *F* is the flux ($erg \cdot s^{-1} \cdot cm^{-2}$) and *L* is the luminosity ($erg \cdot s^{-1}$) and D the distance to the star: $\boxed{F = \frac{L}{4\pi D^2}}$
- Radius (*R*)
- Effective temperature (T_e)
  - The temperature of a black body of the same radius as the star that would radiate the same amount of energy. Thus

    $L = 4\pi R^2 \sigma T_e^4$

  where σ is the Stefan-Boltzmann constant ($5.67\times 10^{-8}\ Wm^{-2}K^{-4}$)

Magnitudes and Colours





The Hertzsprung-Russell diagram

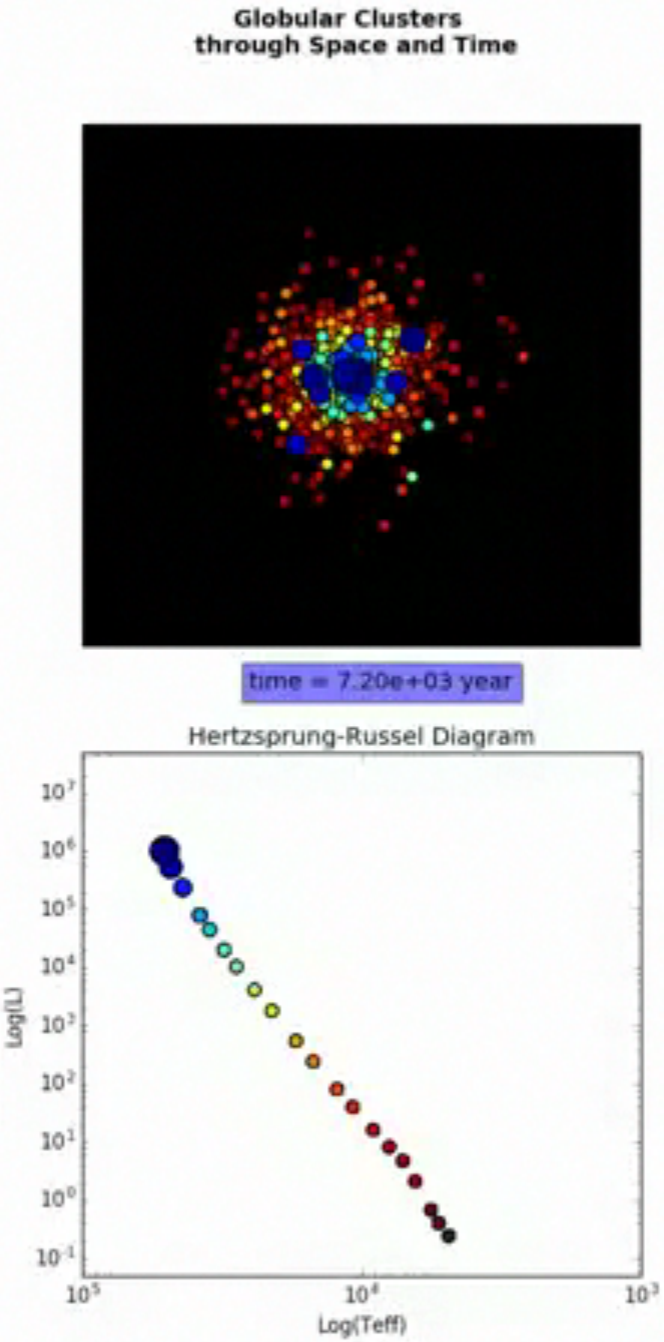
Project carried within the Master SPaCE 2015-2016:

« Globular Clusters through Space & Time »

by

Jon FERNANDEZ OTEGI
Théo LOPEZ
Carla RODRIGUEZ

https://people.lam.fr/burgarella.denis/denis/Master_SPaCE.html




```
# We reset the main parameters of the plot
def update_plot(i, temperature, luminosity,
color, size, elev, azim, dist, scat1, scat2):
    global pause
    # Set colors...
    scat1.set_array(color[i])
    # Set sizes...
    scat1.set_sizes(size[i])
    # Set elevation and azimuth...
    ax1.view_init(elev=elev[i], azim=azim[i])
    # Set distance...
    ax1.dist=dist[i]
    position =
np.column_stack((temperature[i,:],luminosity[i,
:]))
    #print('position', position[i], color[i], size[i])
    scat2.set_offsets(position)
    scat2.set_array(color[i])
    scat2.set_sizes(size[i])

    point.set_data(x, y)
    time_text.set_text(time_template %
(big_array[0,i,0]))

    return scat1, scat2, point, time_text

ani = animation.FuncAnimation(fig,
update_plot, frames=range(numframes),
                    fargs=(temperature_data,
luminosity_data,
                    color_data, size_data,
                    elev_data, azim_data,
dist_data,
                    scat1, scat2), blit=False,
interval=25, repeat=False,
                    init_func=init)

plt.show()
ani.save('Glob_Evol.mp4', fps=24,
extra_args=['-vcodec', 'libx264'])
```

Mass functions

- The stellar masses are one of the most important factors in determining their evolution, so when studying a stellar population, we are interested in estimating their masses.
- An important information is the number of stars per unit mass which is called a mass function.
- We define the mass function *Φ(M)* such that *Φ(M) dM* is the number of stars with masses between *M* and *M + dM*.
- With this definition, the total number of stars with masses between M_{low} and M_{up} is:

$$N(M_{low}, M_{up}) = \int_{Mlow}^{Mup} \Phi(M) dM$$

- By deriving both sides, we get:

$$\frac{dN}{dM} = \Phi(M)$$

- *Φ* is the derivative of the number of stars with respect to mass, i.e. the number of stars *dN* within some mass interval *dM*.

Total mass of stars between M_{low} and M_{up}

- If we are interested in the total mass of stars between M_{low} and M_{up} in a given system rather than the number of such stars, we must integrate Φ times the mass per star. Thus the total mass of stars with masses between M_{low} and M_{up} is:

$$M_*(M_{low}, M_{up}) = \int_{Mlow}^{Mup} M\Phi(M)dM$$

- or equivalently:

$$\frac{dM_*}{dM} = M\Phi(M) = \xi(M)$$

$y = ln(m)$
$dy/dm = 1/m$
$dy = dm/m$
$d[ln(m)] = dm/m$

- ξ *(M)* gives the number of star per *ln(M)*, rather than per number in mass.
- **Physical explanation**:
  - Suppose that $\Phi(M)$ = Cte constant: there are as many stars from 1 − 2 $M_{\odot}$ as there are from 2 − 3 $M_{\odot}$ as there are from 3 − 4 $M_{\odot}$, etc.
  - Instead suppose that $\xi(M)$ = Cte: there are equal numbers of stars in intervals that cover an equal range in logarithm, so there would be the same number from 0.1 − 1 $M_{\odot}$, from 1 − 10 $M_{\odot}$, from 10 − 100 $M_{\odot}$, etc.

- The distribution of initial stellar masses (IMF) might be invariant **[*hot topic*]**.

- If we examine two populations, of galaxies of different sizes, then *Φ(M) = dN/dM* will be different because they have different numbers of stars. However, they may have the same fraction of their stars in a given mass range.

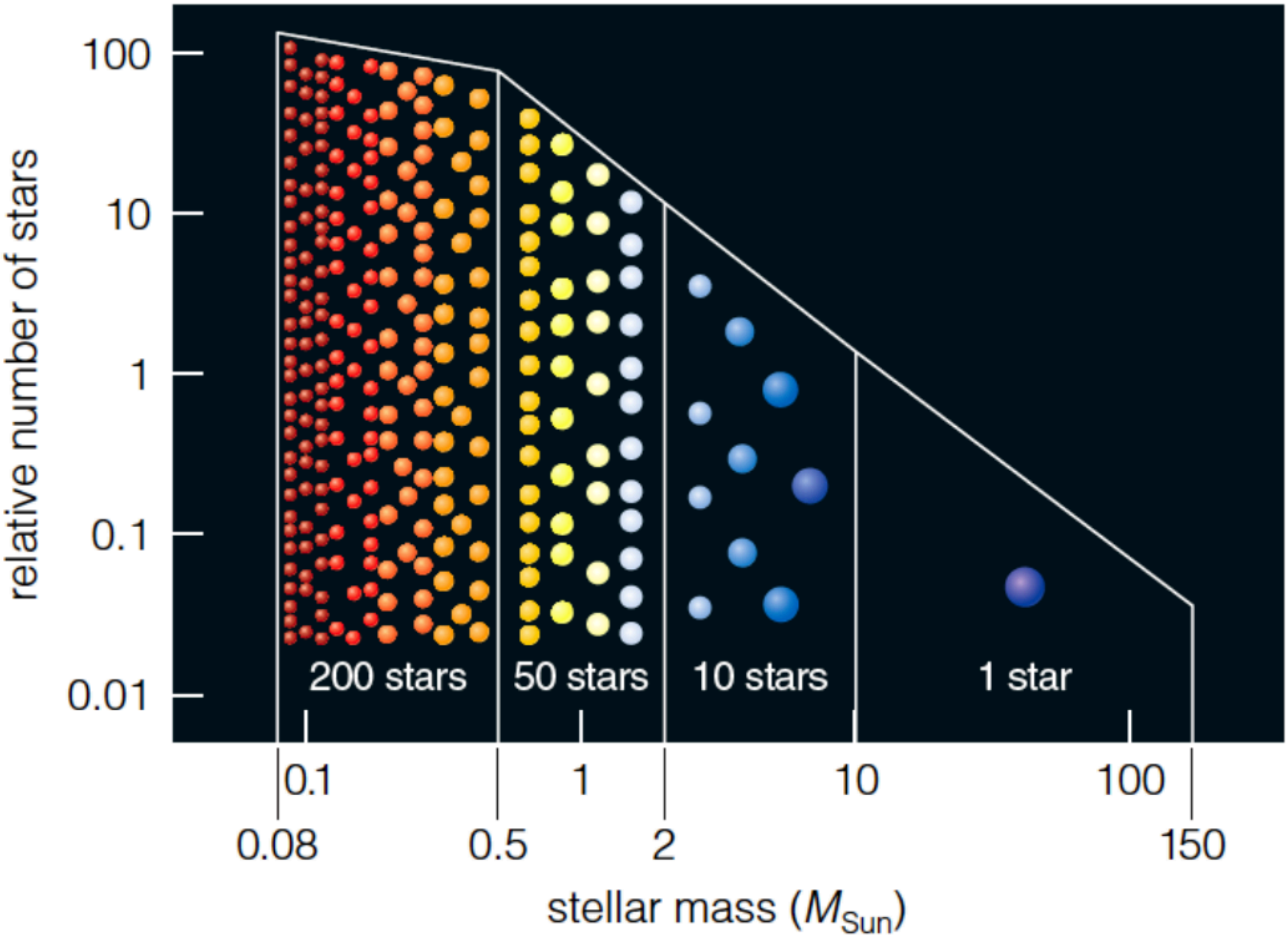
- So, it is generally common to normalize *Φ* or *ξ* so that the integral is unity, i.e. to compute a normalization factor for *Φ* or *ξ* such that the integrals are equal to 1.

- When the mass is normalized, *Φ(M) dM* and *ξ(M) dM* give the fraction of stars (fraction by number for *Φ* and fraction by mass for *ξ*) with masses between *M* and *M + dM*.

$$\int_{m_{\rm low}}^{m_{\rm up}} m\phi(m)dm = \int_{m_{\rm low}}^{m_{\rm up}} \xi(m)dm = 1 \quad \textit{with } m_{low} = 0.1M_\odot \textit{ and } m_{up} = 120M_\odot$$

In LMC at 165 000 l.y.
- R136a1 : 265 $M_\odot$ (?)
- R136a2 : 195 $M_\odot$ (?)

η Carinae : 120 $M_\odot$



100
10
1
0.1
0.01
relative number of stars
200 stars
50 stars
10 stars
1 star
0.1
1
10
100
0.08
0.5
2
150
stellar mass (M_{Sun})

The Salpeter Initial Mass Function (IMF)

If we ignore the low-mass flattening of the IMF below ~ $1M_\odot$, we might assume that the same slope holds over the range $M_{min} = 0.1M_\odot$ *to* $M_{max} = 120M_\odot$.

This is known as the Salpeter IMF: $\phi(M) = \phi_0 \, M^{-2.35}$
The normalization ϕ_0 (in units of $M_\odot$) is evaluated by requiring that the integral equals to 1:

$$1 = \int_{M\,min}^{M\,max} \Phi(M)\,dM = \int_{M\,min}^{M\,max} AM^{-2.35}\,dM = \frac{A}{-1.35}(M_{max}^{-1.35} - M_{min}^{-1.35}) = \frac{A}{1.35}(M_{min}^{-1.35} - M_{max}^{-1.35})$$

$$\boxed{A = \frac{1.35}{M_{min}^{-1.35} - M_{max}^{-1.35}} = 0.060 \; with \; M_{min} = 0.1M_\Theta \; and \; 120M_\Theta}$$

In a similar way, we have in terms of mass:

$$1 = \int_{M\ min}^{M\ max} \xi(M)\,dM = \int_{M\ min}^{M\ max} BM^{-1.35}\,dM = \frac{B}{-0.35}(M_{max}^{-0.35} - M_{min}^{-0.35}) = \frac{B}{0.35}(M_{min}^{-0.35} - M_{max}^{-0.35})$$

$$\boxed{B = \frac{0.35}{M_{min}^{-0.35} - M_{max}^{-0.35}} = 0.17 \; with \; M_{min} = 0.1M_\Theta \; and \; 120M_\Theta}$$

Illustration

So, we will have: $\phi(M) = 0.060\ M^{-2.35}$ and $\xi(M) = 0.17\ M^{-1.35}$ for $M = 0.1 - 120M_{\odot}$

We can estimate the fraction of stars by number and by mass in a given mass range. For instance, what is the fraction of stars by number (1) and by mass (2) more massive than the Sun in a new-born population with a Salpeter IMF?

1) $$f_N(> M_\Theta) = \int_1^{120} 0.060\Phi(M)dM = \int_1^{120} 0.060 M^{-2.35} dM =$$

$$f_N(> M_\Theta) = \frac{0.060}{1.35}(1^{-1.35} - 120^{-1.35}) \boxed{= 0.045}$$

2) $$f_M(> M_\Theta) = \int_1^{120} 0.17 M\Phi(M)dM = \int_1^{120} 0.17 M^{-1.35} dM =$$

$$f_M(> M_\Theta) = \frac{0.17}{0.35}(1^{-0.35} - 120^{-0.35}) = \boxed{0.40}$$

4.5% of the stars are more massive that the Sun but,
the mass of these stars amounts to 40% of the new-born stars.

Two remarks

The total stellar mass of a system is computed from the following integral:

$$M_{tot} = \int_{M\min}^{M\max} M\Phi(M)dM = \int_{M\min}^{M\max} AMM^{-2.35}dM = \int_{M\min}^{M\max} AM^{-1.35}dM$$

$$M_{tot} = \frac{A}{-0.35}(M_{\max}^{-0.35} - M_{\min}^{-0.35}) = \frac{A}{0.35}\boxed{(M_{\min}^{-0.35} - M_{\max}^{-0.35})}$$

It shows that most of the stellar mass lies in low-mass stars.

The total luminosity (assuming $L = C\,M^{3.5}$) is computed as follows:

$$L_{tot} = \int_{M\min}^{M\max} L(M)\Phi(M)dM = \int_{M\min}^{M\max} CM^{3.5}AM^{-2.35}dM = \int_{M\min}^{M\max} ACM^{1.15}dM$$

$$L_{tot} = \frac{AC}{2.15}\boxed{(M_{\min}^{2.15} - M_{\max}^{2.15})}$$

which shows that the total stellar luminosity is dominated by the most massive stars.

Other more realistic IMFs (mainly for faint stars)

IMF Scalo (1998): $\xi(m) = m^{-\alpha}$

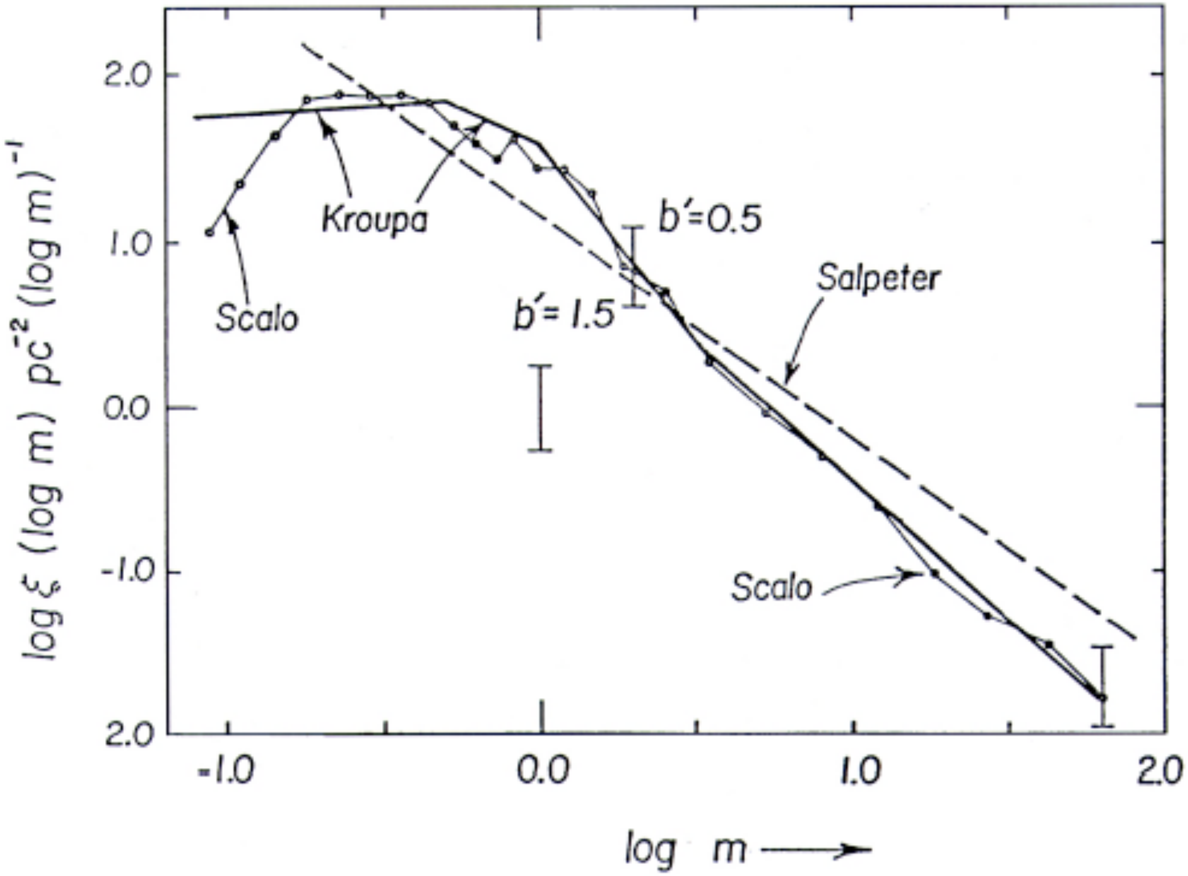
- $\alpha = -0.2 \pm 0.3$ for $0.08 \leq m/M_\odot < 1\ M_\odot$
- $\alpha = -1.7 \pm 0.5$ for $1 \leq m/M_\odot < 10\ M_\odot$
- $\alpha = -1.3 \pm 0.5$ for $10 \leq m/M_\odot < 100\ M_\odot$

IMF Kroupa 2001: $\xi(m) = m^{-\alpha}$

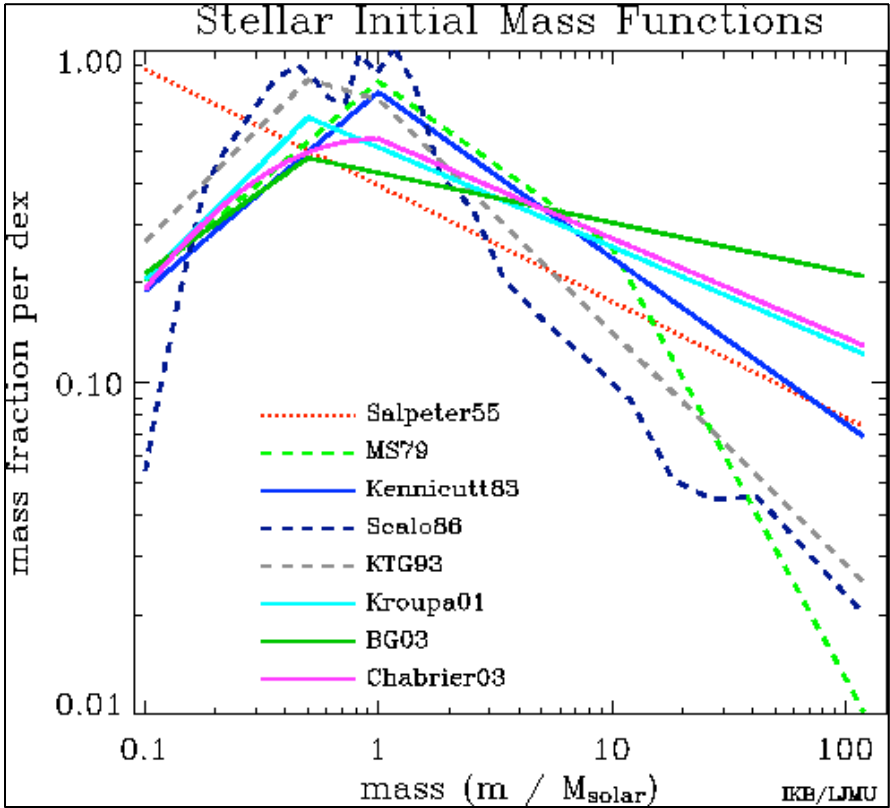
[$\alpha = 0.3$ for $m/M_\odot < 0.08$]

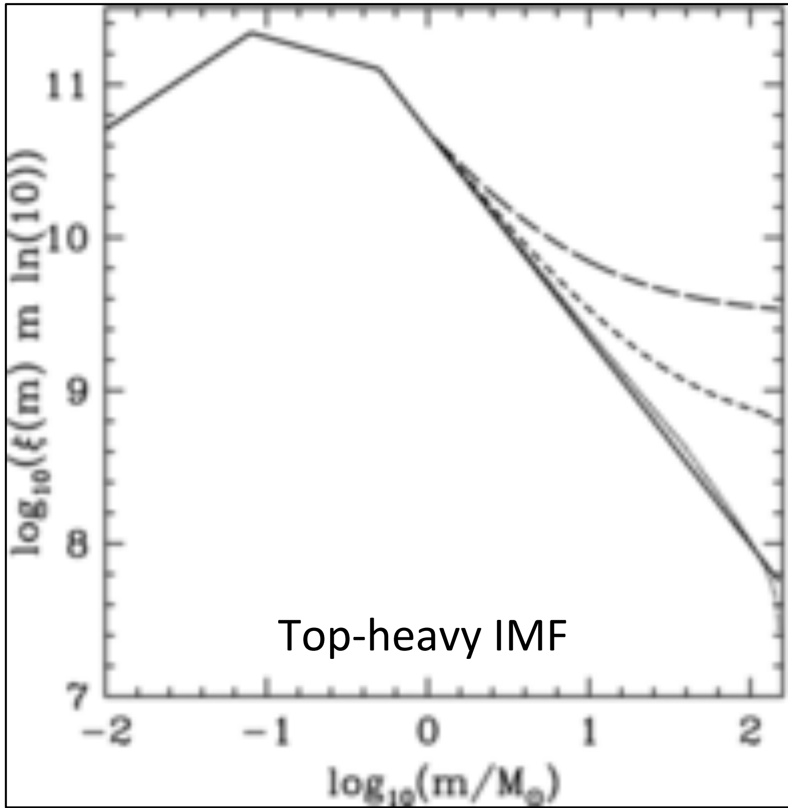
$\alpha = 1.3$ for $0.08 \leq m/M_\odot < 0.5$

$\alpha = 2.3$ for $0.5 \leq m/M_\odot$



Sample Initial Mass Functions of Stars





$$\Phi_{top-heavy}(M) \propto \begin{cases} \Phi_{Salp}(M) \propto M^{-2.35},\ for\ 0.1 M_{\Theta} < M < 100 M_{\Theta} \\ e.g.\ M^{-1},\ for\ 100 M_{\Theta} < M < 500 M_{\Theta} \end{cases}$$

Variability of the IMF ?

An important *if not crucial question* is:

- Is the Initial Mass Function universal or does it vary with the environment, the element abundances (metallicity), the redshift, the star formation rate, the Jeans mass, … ?

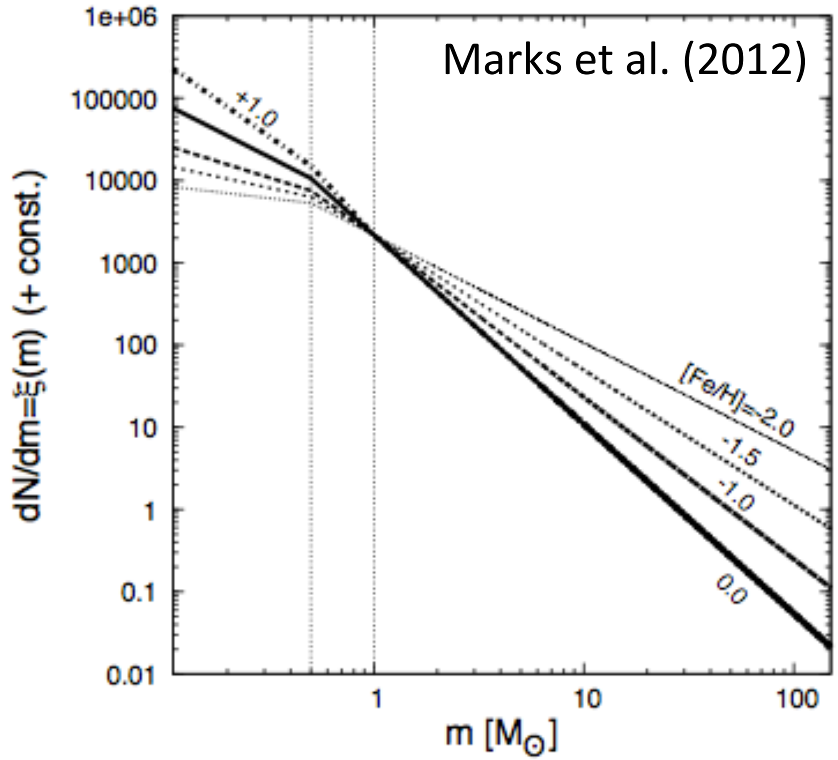


Figure 5. Suggested shape of the stellar IMF for different metallicities, [Fe/H] (not taking into account the density dependence of the IMF). The IMFs are scaled such that their values agree at $m = 1M_\odot$. Above $1M_\odot$ the IMF slope is determined by the present work (Fig. 4, equation 11). Below $1M_\odot$ the parametrisation is by Kroupa (2001, equation 12), whose results suggest tentative evidence that more metal-rich environments produce relatively more low-mass stars. Note that only the metallicity dependence is shown, but not the dependence on mass (Fig. 2) or density (Fig. 3).

Why is the IMF of major importance ?

Several reasons that range from a basic better understanding of the star formation process to wider Xtragalactic / cosmological consequences:

• Changing the IMF => changing the stellar mass (M_*) of the stars in a system (e.g. a galaxy)

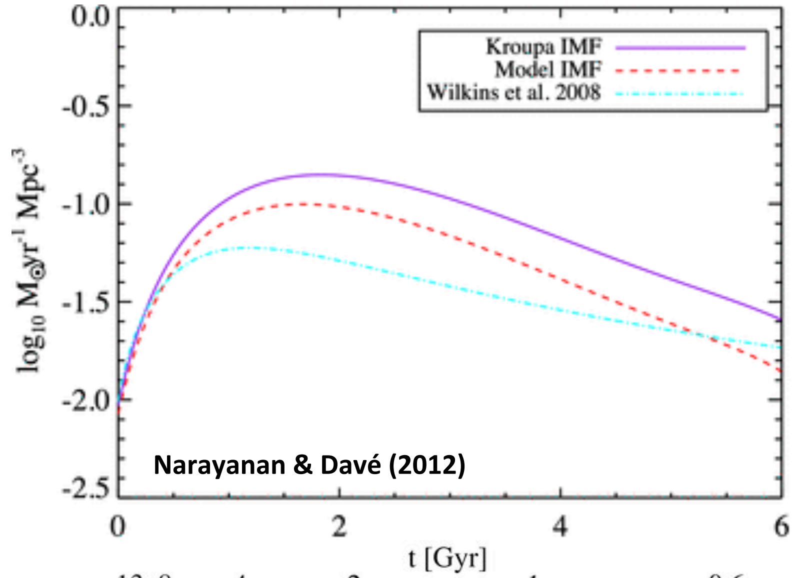
• Changing M_* => changing the star formation rate (SFR = M_* / Δt) and therefore all parameters based on SFR:

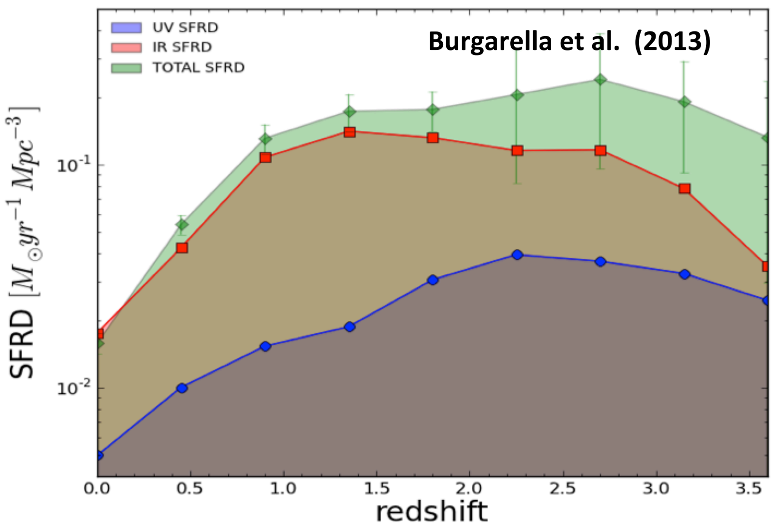
- ○ Cosmic SFR density = density of star formation per unit volume of the universe at a function of redsfhit
- ○ Specific SFR = SFR / M_* => *very* important parameter that provides an estimate of the star formation activity in galaxies

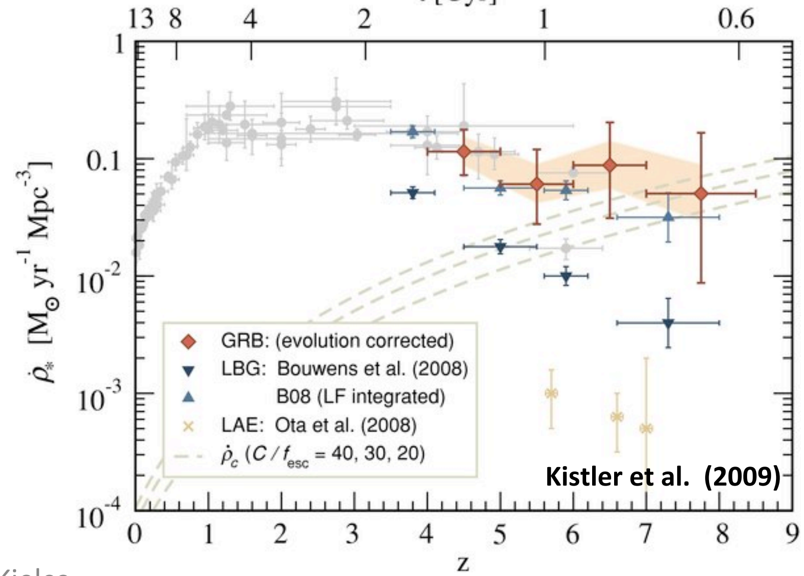
Cosmological implications of a stellar initial mass function that varies with the Jeans mass in galaxies

Evolution of cosmic SFR density.

The purple solid line shows model form from Hopkins et al. (2010), assuming a Kroupa IMF
With a varying IMF model, the SFR density decreases, as is shown by the red dashed line
The blue dash–dotted line shows the Wilkins et al. (2008) SFR density that would be necessary to match the observed present-day stellar mass density

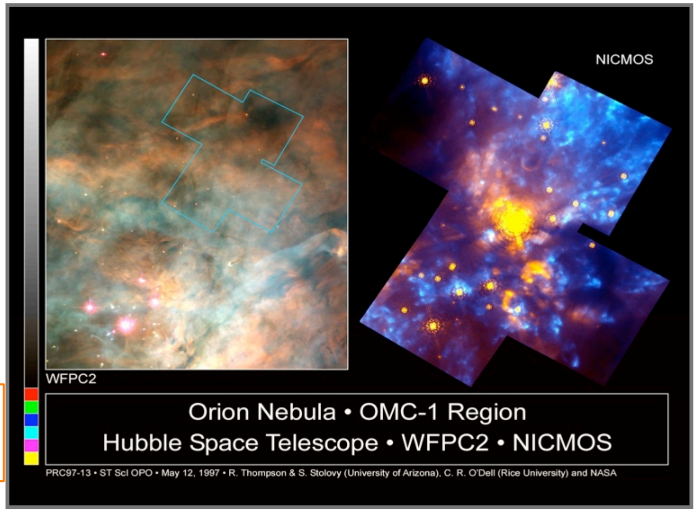






Giant molecular clouds (GMC)

Star formation takes place in cold, dense gas clouds: The molecular clouds. Stars form in groups or clusters. The largest GMC in Orion is about 1000 light years away. Hot young stars (25-50 million year old) ionize their surroundings and are therefore easily visible.

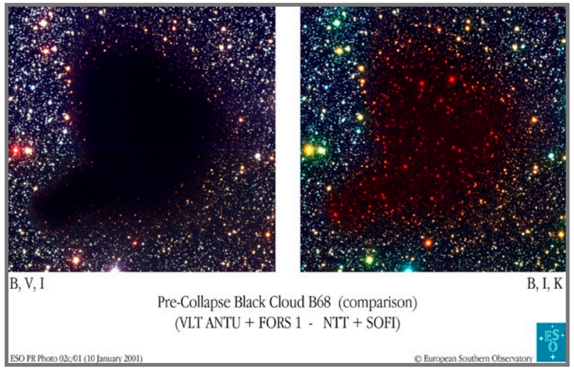


- *main characteristics:*
  - molecular hydrogen
  - ~1% of dust (Si and C)
  - organic and non-organic molecules ammoniac NH_3, formaldehyde H_2CO, acetylene HC_3N
  - ~ 10^5-10^6 solar masses
  - ~ 1000 atoms per cm^3
  - clumpy: 10^3-10^4 solar masses clumps
  - T ≈ 10 K

⇒ Wien law $\lambda_{max} = 2.9 \times 10^{-3} / T \sim 290\ \mu m$
⇒ Observation in Far-IR (Herschel, ALMA, PdB)

- *structure*
  - emission of the H_2 molecule but difficult because small dipole moment. Molecules with larger dipole moment (CO) are better but do not represent the mass...
  - dust emission (radio band). Interpretation difficult since many parameter unknown (departure from LTE, opacity, dust temperature, etc.)
  - IR cameras on large telescopes measure the absorption on thousands of background stars → map the dust distribution in the cloud

Bok globule: Barnard 68



Bok globules
Example characteristics:
- 410 light years away
- diameter ~ 12'500 AU
- T ~ 16 K
- ~ 2.1 solar masses
- in gravitational equilibrium with P_{bound}=12.5 × 10^{-12} Pa

Star Formation Sequence in brief

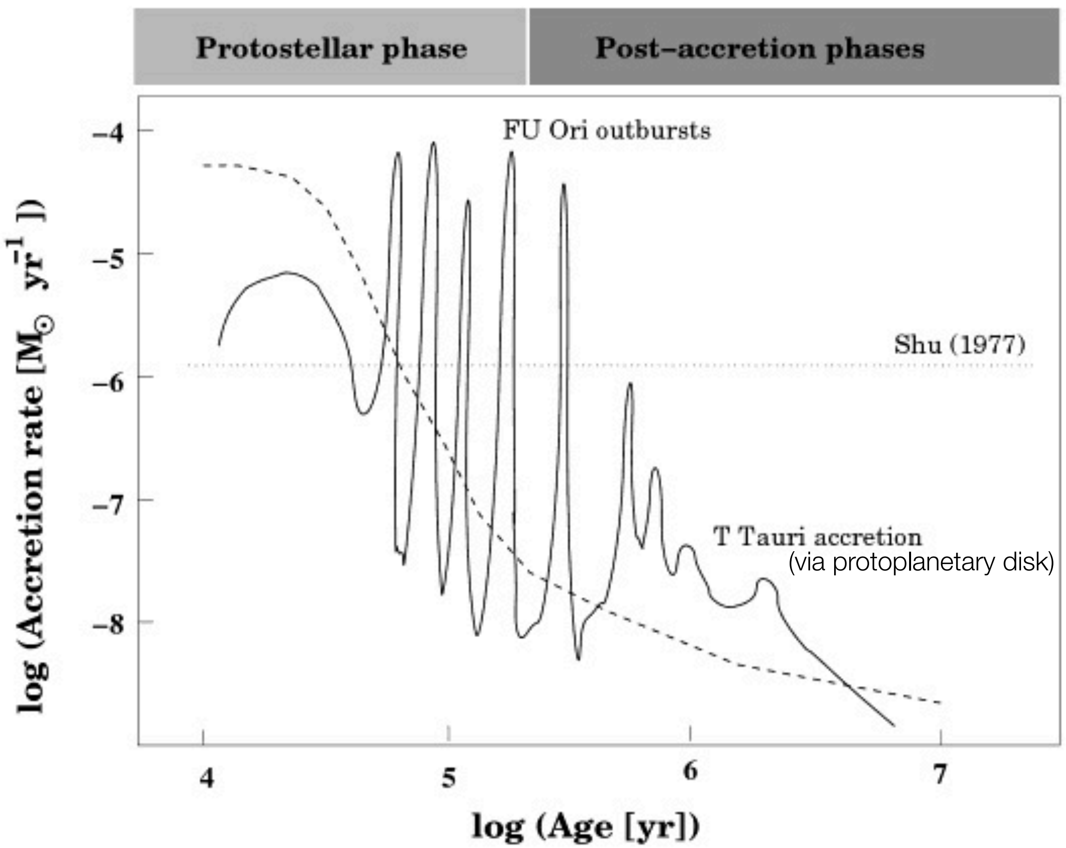
- Jeans instability => gas cloud collapse begins (~ 10^5 M_{sun})
- Isothermal collapse (free-fall time 10^8 years)
- Fragmentation of the cloud of gas
- Center of cloud becomes opticall thick: adiabatic compression (~ 10^{-13} g/cm^3) => thermodynamically isolated i.e., no heat transfered to the surroundings
- First hydrostatic core forms: ~ 170K (first equilibrium phase)
- H_2 dissociation (T ~ 2 000K) and second core collapse
- Ionized H in second core, dynamically stable again (10^{-3} M_{sun}, 20 000K, second equilibrium phase)
- Pre-Main Sequence contraction
- Zero Age Main Sequence (ZAMS): luminosity produced by H fusion, minimum mass 0.08 M_{sun}

Overview star formation sequence: Nomenclature

- Protostar
  - optically thick stellar core
  - forms during the end of the adiabatic contraction phase and grows during the accretion phase.
  - large accretion rates ~10^{-5} M_{sun}/yr

- Pre Main Sequence Star
  - visible in the optical
  - small accretion rates ~10^{-7} M_{sun}/yr
  - energy generation mainly via contraction

- ZAMS: Zero age main sequence
  - PMS contraction => center heats up
  - ~3 10^6 K: H burning ignites
  - contraction stops, energy production mainly via fusion



The virial theorem

The virial theorem is given by: $$\frac{1}{2}\frac{d^2I}{dt^2} = 2E_c + \Omega$$

Where the gravitational and the kinetic energies are accounted.

In numerous cases (slow evolution, quasi-static), the second derivative of the inertial momentum is zero and we have:

$$2E_c + \Omega = 0$$

- However, *we* need to account for all the forces acting on the particles, internal or external to the system:
- Assuming an ideal gas with internal forces that produces an internal pressure $\mathbf{p_{int} = (\gamma\text{-}1)\rho u}$ with the specific heat ratio equal to $\gamma = c_p / c_v$. u is the internal energy density and $\mathbf{p_S}$ the constant surface pressure :

$$\sum_i \overrightarrow{F_i}\,\vec{r}_i = 3(\gamma - 1)\int \rho u dV + \int_S -p_S \vec{n}_{ext} . \vec{r} dS = 3(\gamma - 1)U_{therm} - p_S \int_V \nabla r . dV$$

$$\sum_i \overrightarrow{F_i}\,\vec{r}_i = 3(\gamma - 1)U_{therm} - 3p_S V$$

$$\nabla r = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1 + 1 + 1 = 3$$

$$-p_S \int_V \nabla r . dV = -3p_S \int_V dV = -3p_S V$$

- Finally:

$$\boxed{\frac{d^2 I}{dt} = 2E_c + \Omega - 3P_S V + 3(\gamma - 1)U_{therm}}$$

γ = 5/3 for an ideal gas
c_p: heat capacity at constant pressure
c_v: heat capacity at constant volume

- From the previous equation: $\frac{d^2I}{dt} = 2E_c + \Omega - 3P_S V + 3(\gamma - 1)U_{therm}$
- Let's assume a static configuration with no external pressure:

$$\frac{d^2I}{dt} = 2E_c + \Omega + 3(\gamma - 1)U_{therm}$$

- In absence of any internal energy, we find the same equation as before:

$$2E_c + \Omega = 0$$

- In the absence of any bulk kinetic energy (e.g. not rotation):

$$\Omega + 3(\gamma - 1)U_{therm} = 0$$

- Let's define: $E_{tot} = \Omega + U_{therm}$
- Then: $\Omega = E_{tot} - U_{therm}$ and $U_{therm} = E_{tot} - \Omega$

- So from: $\Omega + 3(\gamma - 1)U_{therm} = 0$

- We have:
$$E_{tot} - U_{therm} + 3(\gamma - 1)U_{therm} = 0$$
$$E_{tot} = (1 - 3\gamma + 3)U_{therm} = (4 - 3\gamma)U_{therm}$$
$$\boxed{E_{tot} = (4 - 3\gamma)U_{therm}}$$

- And:
$$\Omega + 3(\gamma - 1)(E_{tot} - \Omega) = 0$$
$$\Omega + 3\gamma E_{tot} - 3E_{tot} - 3\gamma\Omega + 3\Omega$$
$$3(\gamma - 1)E_{tot} = -\Omega - 3\Omega + 3\gamma\Omega$$
$$\boxed{E_{tot} = \frac{(3\gamma - 4)}{3(\gamma - 1)}\Omega}$$

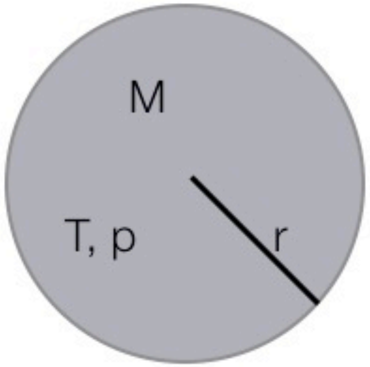
- Which is an example of a non-rotating star in hydrostatic equilibrium.

f is the number of degrees of freedom: f = 3 for mono-atomic and f = 5 for diatomic

- Remarks:
  - $\gamma > 4/3 \Rightarrow E_{tot} < 0$ and the system is bound. For instance for a mono-atomic gas, $\gamma = (f+2) / f = 5/3$
    $E_{tot} = \frac{1}{2}\Omega = -U_{therm}$ i.e. if the contraction is slow and quasi-static, **half of the potential energy is transformed in internal energy (heat)** and the rest is lost in radiation. The system is not stable.
  - $\gamma = 4/3 \Rightarrow E_{tot} = 0$, i.e., the gas is radiation dominated and the total energy is independant of the radius.
  - $\gamma < 4/3 \Rightarrow E_{tot} > 0$ i.e the system is not bound and the system is stable.

Jeans instability

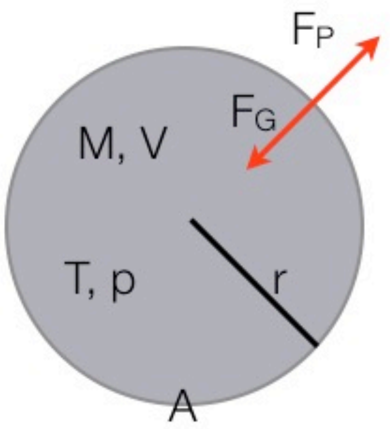
Now we want to understand under which conditions small perturbations of a gas cloud grow exponentially, leading to the collapse of the cloud (and, in the end star formation). The so-called Jeans instability describes the gravitational instability of a self-gravitating gas cloud. There are several ways to derive such a criterion, with increasing complexity.



The initially stable, static cloud can get initially compressed a bit (perturbed) by a shock wave due to a nearby supernova, passing spiral arms of the galaxy, ..

Jeans instability from force balance

The simplest, order of magnitude estimation can be obtained by force balance arguments.



Collapse occurs if the inwards directed gravitational force is bigger than the outwards directed pressure force.

Consider mean unit forces per volume.

$$\frac{F_P}{V} \sim \frac{F_P}{Ar} \sim \frac{p}{r} \qquad \frac{F_G}{V} \sim \frac{GM^2}{r^2 V} \sim \frac{GM\rho}{r^2} \sim G\rho^2 r$$

Force balance $\frac{p}{r} = G\rho^2 r$ so $r = \sqrt{\frac{p}{G\rho^2}}$ critical maximal radius that allows stability.

We note:

$r \propto \sqrt{p}$ higher pressure=>more stability

$r \propto \frac{1}{\rho}$ higher density=>less stability

with $p = \frac{k}{\mu m_H}\rho T \; = c^2\rho$

$$\boxed{r = \frac{c}{\sqrt{G\rho}}}$$

dense, cold clouds are unstable

Jeans instability from the virial theorem

Let us again assume a static, spherical, homogenous cloud of radius r. We now add the fact that the cloud is in equilibrium with an outside medium at a pressure p_{ext}. We further assume that the cloud does not have any bulk kinetic energy and the the pressure is constant throughout the cloud.

We recall that the general virial theorem is given as

$$\frac{d^2 I}{dt^2} = 2E_{kin} + 3(\gamma - 1)U_{therm} + W - 3p_S V$$

I: moment of inertia

With our assumptions this becomes

$$3(\gamma - 1)U_{therm} + W - 4\pi p_{ext} r^3 = 0 \qquad 3(\gamma - 1)\frac{f}{2}NkT - q\frac{GM^2}{r} - 4\pi p_{ext} r^3 = 0$$

where we have assumed an ideal gas with f degrees of freedom. The factor q in the gravitational potential energy depends on the density distribution. For a homogeneous density, it is 3/5

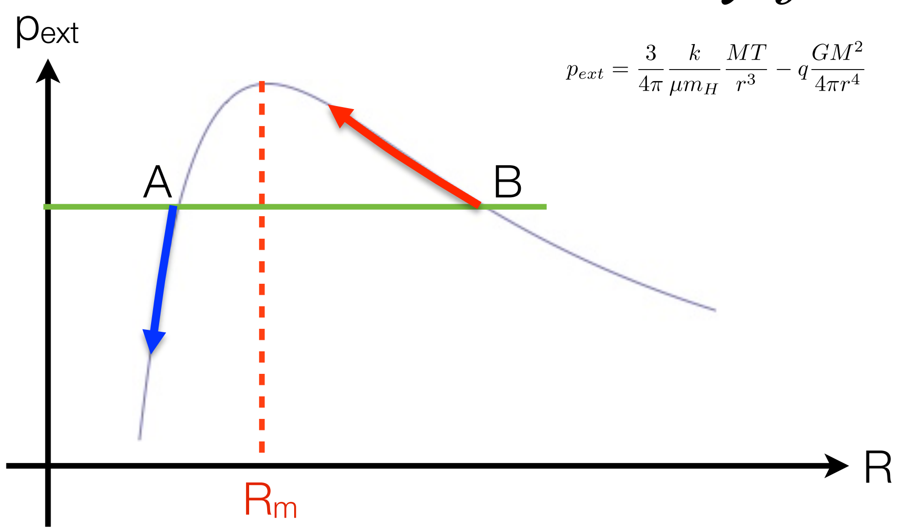
Assuming further f=3 (1 atomic gas) and $N = M/(\mu m_H)$ γ = 5/3 for an ideal gas

$$\boxed{p_{ext} = \frac{3}{4\pi}\frac{k}{\mu m_H}\frac{MT}{r^3} - q\frac{GM^2}{4\pi r^4}}$$

We first note that we can use the ideal gas law $p = \frac{k}{\mu m_H}\rho T$, so $p_{ext} = p - q\frac{GM^2}{4\pi r^4}$

This tells us that the pressure in the cloud must be higher in the presence of gravity. Or in other words, the necessary external pressure to confine the gas is lower. In its absence, the pressure in the cloud and the one outside are the same.

Jeans instability from the virial theorem II



$$p_{ext} = \frac{3}{4\pi}\frac{k}{\mu m_H}\frac{MT}{r^3} - q\frac{GM^2}{4\pi r^4}$$

For fixed M and T the external pressure has the following shape as a function of R. The curve gives us possible equilibrium states (radii) at a specified external pressure.

$$p_{ext} = p - q\frac{GM^2}{4\pi r^4}$$

1) At point B, if we decrease R (compression), p_{ext} increases (and with it also the internal p). This is a stable equilibrium.

2) At point A, if we decrease R (compression), p_{ext} necessary to confine the cloud decreases too (and with it also the internal p). This is clearly an unstable equilibrium.

We can find the critical radius R_m which divides the two regimes by setting $\frac{\partial p_{ext}}{\partial r} = 0$ *(see next page for demonstration)*

which yields

$$\boxed{R_m = \frac{4q}{9}\frac{\mu m_H}{k}\frac{GM}{T}}$$

$$3(\gamma-1)\frac{f}{2}NkT - q\frac{GM^2}{r} = 4\pi p_{ext} r^3$$

$$\Rightarrow p_{ext} = \frac{3\frac{2}{3}\frac{3}{2}\frac{M}{\mu m_H}kT - \frac{3}{5}\frac{GM^2}{r}}{4\pi r^3} \text{ with } \begin{cases} \text{gas mono-atomic: } \gamma = \frac{5}{3} \text{ or } \gamma - 1 = \frac{2}{3} \\ \mathrm{N} = \frac{\mathrm{M}}{\mu \mathrm{m_H}} \\ \mathrm{q} = \frac{3}{5} \text{ for an homogeneous distribution} \end{cases}$$

$$p_{ext} = \frac{3}{4\pi}\frac{k}{\mu m_H}\frac{MT}{r^3} - \frac{3}{20\pi}\frac{GM^2}{r^4} = 0 = \frac{A}{r^3} + \frac{B}{r^4} = Ar^{-3} + Br^{-4} \text{ and } r \neq 0$$

The variation presents an extremum if $\frac{\partial p_{ext}}{\partial r} = 0$

$$\frac{\partial p_{ext}}{\partial r} = 0 \Leftrightarrow -3Ar^{-4} - 4Br^{-5} = -r^{-4}(3A + 4Br^{-1}) = 0 \text{ or } 3A + 4Br^{-1} = 0 \text{ and } r \neq 0$$

$$r_{\max} = \frac{-4B}{3A} = \frac{-4(-\frac{3}{20\pi}GM^2)}{3\frac{3}{4\pi}\frac{k}{\mu m_H}MT} = \frac{\frac{3}{5\pi}GM^2}{\frac{9}{4\pi}\frac{k}{\mu m_H}MT} = \frac{\frac{3}{5}GM}{\frac{9}{4}\frac{k}{\mu m_H}T} = \boxed{\frac{qGM}{\frac{9}{4}\frac{k}{\mu m_H}T}}$$

$$r_{\max} = \frac{qGM}{\frac{9}{4}\frac{k}{\mu m_H}T} = \frac{4q}{9}\frac{\mu m_H}{k}\frac{GM}{T} \text{ with } M = \frac{4}{3}\pi\rho r_{\max}^3 \text{ and } p = \frac{k}{\mu m_H}\rho T = c^2\rho \Rightarrow c^2 = \frac{kT}{\mu m_H}$$

$$r_{\max} = \frac{4q}{9}\frac{\mu m_H}{k}\frac{GM}{T} = \frac{4q}{9}\frac{1}{c^2}G\frac{4}{3}\pi\rho r_{\max}^3 \Rightarrow \frac{r_{\max}}{r_{\max}^3} = \frac{16}{27}\frac{3}{5}\frac{G}{c^2}\pi\rho \Rightarrow r_{\max}^2 = \frac{45}{16}\frac{c^2}{G\pi\rho}$$

$$\boxed{r_{\max} = \sqrt{\frac{45}{16\pi}}\frac{c}{\sqrt{G\rho}} = \lambda_J}$$ <u>Jeans length</u>

$$M_J = \frac{4}{3}\pi\rho r_{\max}^3 = \frac{4}{3}\pi\rho\left(\frac{45}{16\pi}\right)^{\frac{3}{2}}\frac{c^3}{(G\pi\rho)^{\frac{3}{2}}} = \sqrt{\frac{45}{16\pi}}\frac{15}{4}\frac{c^3}{G^{\frac{3}{2}}\rho^{\frac{1}{2}}} = \sqrt{\frac{5}{\pi\rho}}\frac{45}{16}\frac{c^3}{G^{\frac{3}{2}}} = \sqrt{\frac{5}{\pi\rho}}\frac{45}{16}\left(\frac{c^2}{G}\right)^{\frac{3}{2}}$$

$$\boxed{M_J = \sqrt{\frac{5}{\pi\rho}}\frac{45}{16}\left(\frac{kT}{G\mu m_H}\right)^{\frac{3}{2}}}$$ <u>Jeans mass</u>

$$\text{and finally: } M_J = \rho^{-\frac{1}{2}}T^{\frac{3}{2}}\left(\frac{5k}{G\mu m_H}\right)^{\frac{3}{2}}\frac{3^2}{4^2\sqrt{\pi}}$$

During collapse, the density increases and therefore M_J decreases, which leads to fragmentation of the cloud.

Jeans instability from the virial theorem III

The Jeans length is from this analysis

$$\boxed{\lambda_J = \sqrt{\frac{45}{16\pi}} \frac{c}{\sqrt{G\rho}}} \propto \sqrt{\frac{1}{\rho}}$$

The numerical constant is very close to unity (ca. 0.95), so we find a similar result as before.

The corresponding Jeans mass is

$$\boxed{M_J = \frac{45}{16} \sqrt{\frac{5}{\pi\rho}} \left(\frac{kT}{G m_H \mu} \right)^{3/2}} \approx 4.89 \times 10^{21} \left(\frac{T}{\mu} \right)^{3/2} \rho^{-1/2} \ [\mathrm{kg}] \propto \sqrt{\frac{T^3}{\rho}}$$

Using typical values $\mu = 2.35$ $T = 10\mathrm{K}$ $\rho = 10^{-19}$ kg / m^3

we find $M_J \approx 69 M_\odot$ and $\lambda_J \approx$ 3.9 pc ≈ 800 000 AU

The collapse thus begins with large masses.

This is more than the typical mass of a single star (less than 1 M_{sun}). This indicates that during the collapse, only part of the gas ends up in stars, and that the cloud fragments during collapse. Thus many stars form out of one collapsing cloud, which means that young stars get born in clusters.

G = 6.67 x 10^{-11} m^3 kg^{-1} s^{-2}, k_B = 1.38 x 10^{-23} J K^{-1} and m_H = 1.67 x 10^{-27} kg

Collapse of a T = 0 K spherical homogeneous gas cloud

Collapse of a T=0 K spherical homogeneous gas cloud

We now consider what is happening after the collapse has started. We assume that pressure does not play a role (yet), which we (formally) can represent by assuming T=0 K.

Then, the equation of motion of a spherical cloud is simply given as

$$\frac{d^2r}{dt^2} = \frac{dr}{dt}\frac{dv}{dr} = v\frac{dv}{dr} = -\frac{GM(r)}{r^2}$$

Integration by separation of the variables with the initial conditions v(0)=0 and r(0)=r_0 yields the velocity as a function of r

$$v(r) = \sqrt{2GM(r)\left(\frac{1}{r} - \frac{1}{r_0}\right)} = \frac{dr}{dt}$$

Next we look for v(t) and r(t). With the ansatz* r=$r_o \cos^2(\alpha)$ we eventually find the non algebraic equation for α(t)

$$\boxed{\alpha + \frac{1}{2}\sin(2\alpha) = \left(\frac{2GM}{r_0^3}\right)^{1/2} t}$$

Quickly, let's see how we get this.

* educated guess

The acceleration $a = \frac{d^2r}{dt^2} = \frac{d}{dt}\frac{dr}{dt} = \frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = v\frac{dv}{dr} = -\frac{GM}{r^2} \Rightarrow vdv = -\frac{GM}{r^2}dr$

$$d(v^2) = vdv + vdv = 2vdv = -2\frac{GM}{r^2}dr$$

$$v^2(r) = -2GM\int_{r_0}^{r}\frac{dr}{r^2} = -2GM\left[\frac{1}{r}\right]_{r_0}^{r} = 2GM\left(\frac{1}{r}-\frac{1}{r_0}\right) \Rightarrow v(r) = \sqrt{2GM\left(\frac{1}{r}-\frac{1}{r_0}\right)} = \frac{dr}{dt}$$

$$\boxed{v(r) = \sqrt{2GM\left(\frac{1}{r}-\frac{1}{r_0}\right)} = \frac{dr}{dt}}$$

We assume: $r = r_0\cos^2\alpha$

$$\frac{dr}{dt} = -2r_0\cos\alpha.\sin\alpha\frac{d\alpha}{dt} = -\sqrt{\frac{2GM}{r_0}\left(\frac{1}{\cos^2\alpha}-1\right)}$$

$$2\cos\alpha.\sin\alpha\frac{d\alpha}{dt} = \sqrt{\frac{2GM}{r_0^3}\left(\frac{1}{\cos^2\alpha}-\frac{\cos^2\alpha}{\cos^2\alpha}\right)} = \sqrt{\frac{2GM}{r_0^3}\left(\frac{1-\cos^2\alpha}{\cos^2\alpha}\right)} = \sqrt{\frac{2GM}{r_0^3}}\sqrt{\left(\frac{\sin^2\alpha}{\cos^2\alpha}\right)} = \sqrt{\frac{2GM}{r_0^3}}\tan\alpha$$

$$\frac{2\cos\alpha.\sin\alpha\, d\alpha}{\tan\alpha} = \sqrt{\frac{2GM}{r_0^3}}dt \Rightarrow 2\cos^2\alpha\, d\alpha = \sqrt{\frac{2GM}{r_0^3}}dt \Rightarrow \alpha + \sin\alpha\cos\alpha = \boxed{\alpha + \frac{1}{2}\sin 2\alpha = t\sqrt{\frac{2GM}{r_0^3}}}$$

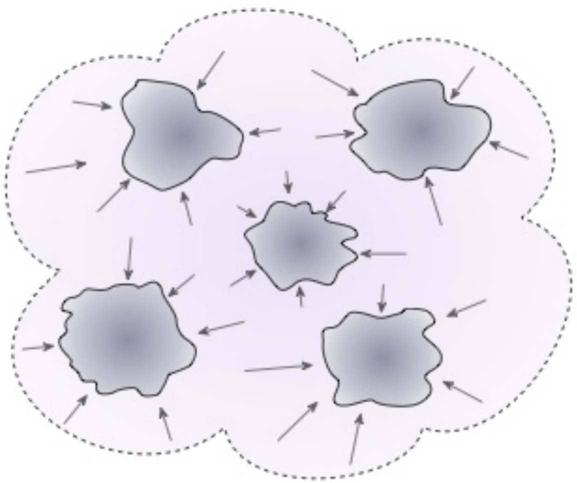
with: $\cos^2 x = \frac{1}{2}\cos 2x + \frac{1}{2} = \frac{1}{2}(1+\cos 2x)$ and $\sin x\cos x = \frac{1}{2}\sin 2x$

Fragmentation I

During the collapse, ρ increases. As long as the density still remains adequately low for the cloud to be transparent, the released thermal energy is radiated into the universe and the temperature remains approximately constant. As

$$M_J \propto \sqrt{\frac{T^3}{\rho}}$$

suggests, this leads to a decrease of the Jeans mass. In particular, sub-sections of the cloud suddenly surpass their own Jeans limit and start collapsing on their own. As also t_{ff} is smaller for higher densities, these sub-collapses proceed faster. This clearly leads to fragmentation.

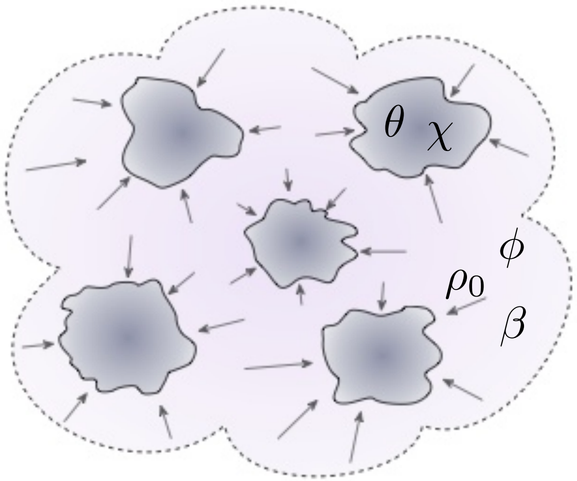


Here we calculate the collapse of an density perturbation in the otherwise homogenous cloud

$$\rho_1 = \rho_0 + \delta\rho$$

Such places might be the origin of later individual star formation, as they decouple.

Fragmentation II



Using the free-fall velocity, we can re-write the master equation:

$$\alpha + \frac{1}{2}\sin 2\alpha = t\sqrt{\frac{2GM}{r_0^3}} = t\sqrt{\frac{2G}{r_0^3}\frac{4\pi}{3}\rho_0 r_0^3} = t\sqrt{\frac{8\pi G\rho_0}{3}} = t\sqrt{\frac{32\pi^2 G\rho_0}{12\pi}}$$

$$= t\frac{\pi}{2}\sqrt{\frac{32G\rho_0}{3\pi}} = \frac{t}{t_{ff}}\frac{\pi}{2} \text{ with } t_{ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{\frac{1}{2}}$$

The collapse occurs at $\alpha=\pi/2$. To compute the difference in collapse between the background cloud and the perturbation, we define β as the parameter characterizing the cloud, and θ the parameter characterizing the perturbation.

Because most of the action happens shortly before the complete collapse, it is convenient to introduce small angles measuring the difference to the full collapse (where we can use that for small angles e.g. $\sin(\alpha)\approx\alpha$) :

$$\beta = \frac{\pi}{2} - \phi \qquad \text{for the background cloud}$$

$$\theta = \frac{\pi}{2} - \chi \qquad \text{for the perturbation}$$

Both angles are <<1, and complete collapse occurs when they vanish.

Inserting this into the master equation, we find after some algebra:

$$\frac{\chi^3}{\phi^3} = 1 - \frac{3\pi}{8\phi^3}\frac{\delta\rho_0}{\rho_0}$$

Fragmentation III

This means that at moment when the perturbation has fully collapsed (χ=0), we have in the background cloud:

$$\phi \approx \left(\frac{3\pi}{8} \frac{\delta\rho_0}{\rho_0} \right)^{1/3}$$

This corresponds to a density increase in the background cloud equal

$$\frac{\rho}{\rho_0} = \left(\frac{r_0}{r}\right)^3 = \frac{r_0^{\,3}}{\left(r_0 \cos^2\beta\right)^3} = \frac{1}{\cos^6\beta} = \frac{1}{\cos^6\left(\frac{\pi}{2} - \phi\right)} = \frac{1}{\sin^6\phi} \approx \frac{1}{\phi^6} = \left(\frac{8}{3\pi}\frac{\rho_0}{\delta\rho_0}\right)^{\frac{6}{3}} = \left(\frac{8}{3\pi}\frac{\rho_0}{\delta\rho_0}\right)^2$$

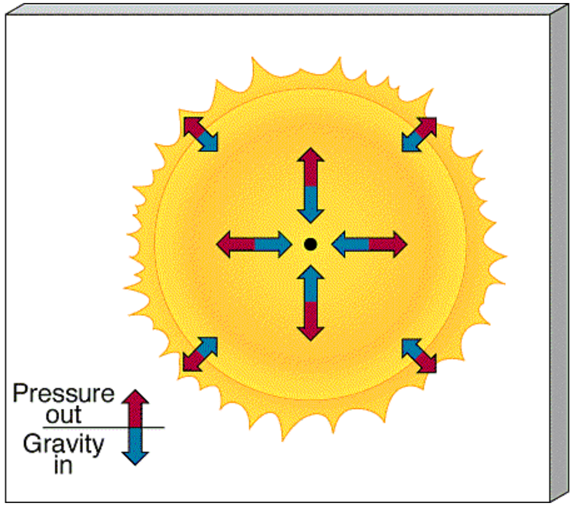
$$\frac{\rho}{\rho_0} = \left(\frac{8}{3\pi}\frac{1}{0.01}\right)^2 \approx \left(\frac{800}{3\pi}\right)^2 \approx 80^2 = 6400$$

For example, an initial density perturbation of 1% ($\delta\rho_0/\rho_0 = 0.01$)will fragment out of the gas cloud when the mean density of the latter has increased by a factor 6400. Keeping in mind that the full density increase form a GMC to a star is ~20 orders of magnitude, this is a small increase only.

In other words, small initial density fluctuations lead to a much faster collapse.

This is important for the formation of stars but also in the early universe.

- Star formation is governed by two dominant influences:
  - gravity, the universal force that causes all matter to attract
  - heat.
- Gravity's pull overcomes the random gas motions within an interstellar cloud, initiating a contraction phase that will last approximately 100,000 years and culminate in the formation of a star.



- During this collapse, the gas density increases. Collisions between atoms and molecules become more frequent and the gas temperature rises.

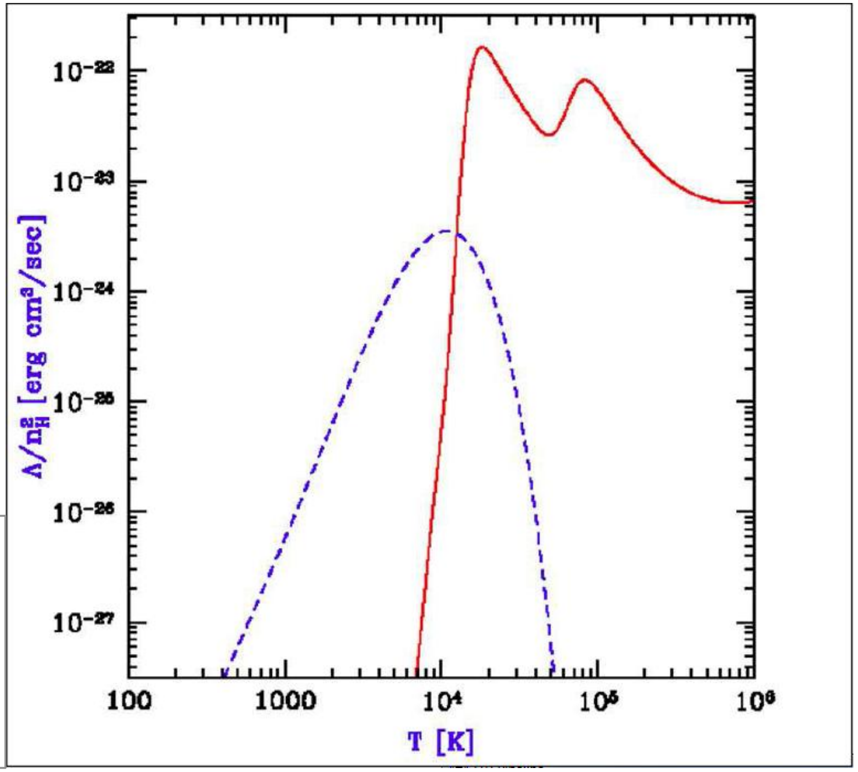
- The heating of the collapsing cloud poses a significant problem: a heated gas wants to expand, the cloud collapse could be halted or even reversed unless heat is effectively and continuously removed from the cloud.

- One process which provides significant cooling involves collisions between molecules.
- When two molecules collide, they convert some of their thermal (kinetic) energy into a form of potential energy. The energy can be stored in the molecule either by simple rotation or by internal vibration or even by lifting one or more electrons into a "higher" less bound orbit around the atoms in the molecule.
- This energy can be later released by the emission of a photon of a particular energy that is characteristic of these molecular species. Photons that escape the cloud carry this energy with them, thus helping to cool the cloud.
- Atoms and molecules are considered to be good coolants if
    - they quickly emit photons following a collision
    - they are present in large enough quantities that a significant number of photons are emitted.
- In this way the collapse of an interstellar cloud is tied to the chemical composition of that cloud.

• Hydrogen and helium are, by far, the most abundant elements in interstellar clouds.

• However, H and He are very poor coolants because they cannot be collisionally induced to emit photons at the low gas temperatures characteristic of molecular clouds.

• A large fraction of the total cooling is produced by a few other atoms and molecules, notably gaseous water (HO), carbon monoxide (CO), molecular oxygen (O), and atomic carbon (C).



Cooling rate of primordial gas as a function of temperature. The solid line represents the contribution from atomic hydrogen and helium and the dashed line represents the contribution from molecular hydrogen. At temperatures below 10^4 K cooling is provided by H2, which is a poor coolant, but at T> 10^4 K more efficient atomic hydrogen line cooling comes to play. Courtesy: Bromm (2012)

From: http://www.cfa.harvard.edu/swas/science1.html

Cooling

- Collisional excitation followed by the emission of an IR photon

Due to their thermal velocity, molecules collide all the time, which can lead to an excitation of an electron. At disexcitation, a photon is radiated, taking away the energy. (We assume here that the cloud is optically thin).

$$A + B \rightarrow A^* + B$$
$$A^* \rightarrow A + h\nu$$

-Frequent collisions (abundant partners)
-Excitation energy comparable to or less than thermal kinetic energy
-High probability of excitation during collision
-Photon emission before the next collision
-No re-absorption of the photon (low optical depth of gas to line emission)

Heating and cooling III

Cooling is in general described by a cooling function $\Lambda(T)$. Its exact value depends on the detailed chemical composition, but an order of magnitude can be estimated from the reaction rate $<\sigma v>$ multiplied by the amount of energy lost in one collision. Taking $\sigma \approx 10^{-16}$ cm^2, $v \approx 1$ km/s, $\Delta E \approx 0.1$ eV $\approx 10^{-13}$ erg, we obtain a typical value of the cooling function: $\Lambda(T) \approx 10^{-24}$ erg / (cm^3 s).

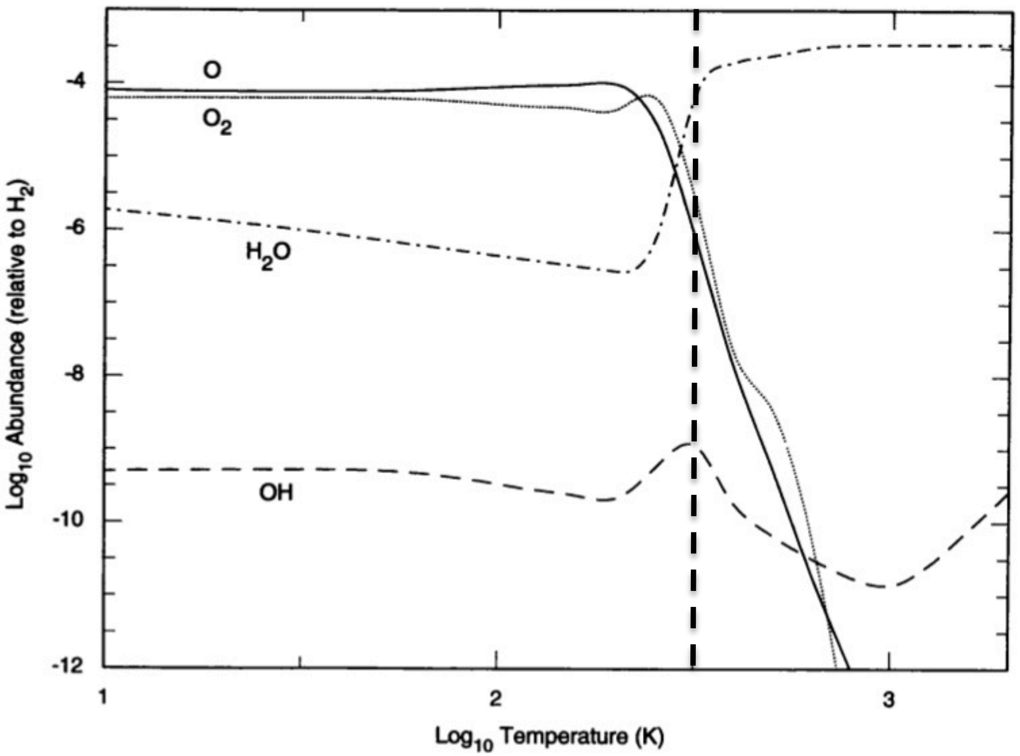
The cooling rate is then obtained by multiplying the cooling function with the abundance of booth cooling species.

$$\Gamma_{CE} = \Lambda(T) n_1 n_2 =< \sigma v > \Delta E \; n_1 n_2 = \sigma \sqrt{\frac{8k\bar{T}}{\pi \bar{m}}} \Delta E \; \frac{\rho_1}{m_1} \frac{\rho_2}{m_2}$$

The cooling time can be estimated (for identical species)as $\tau_{cool} = \dfrac{3/2nkT}{n^2 \Lambda}$

Coolants

To calculate the specific cooling rate, one must know the chemical composition of the gas.



Neufeld et al. 1995

Chemical networks

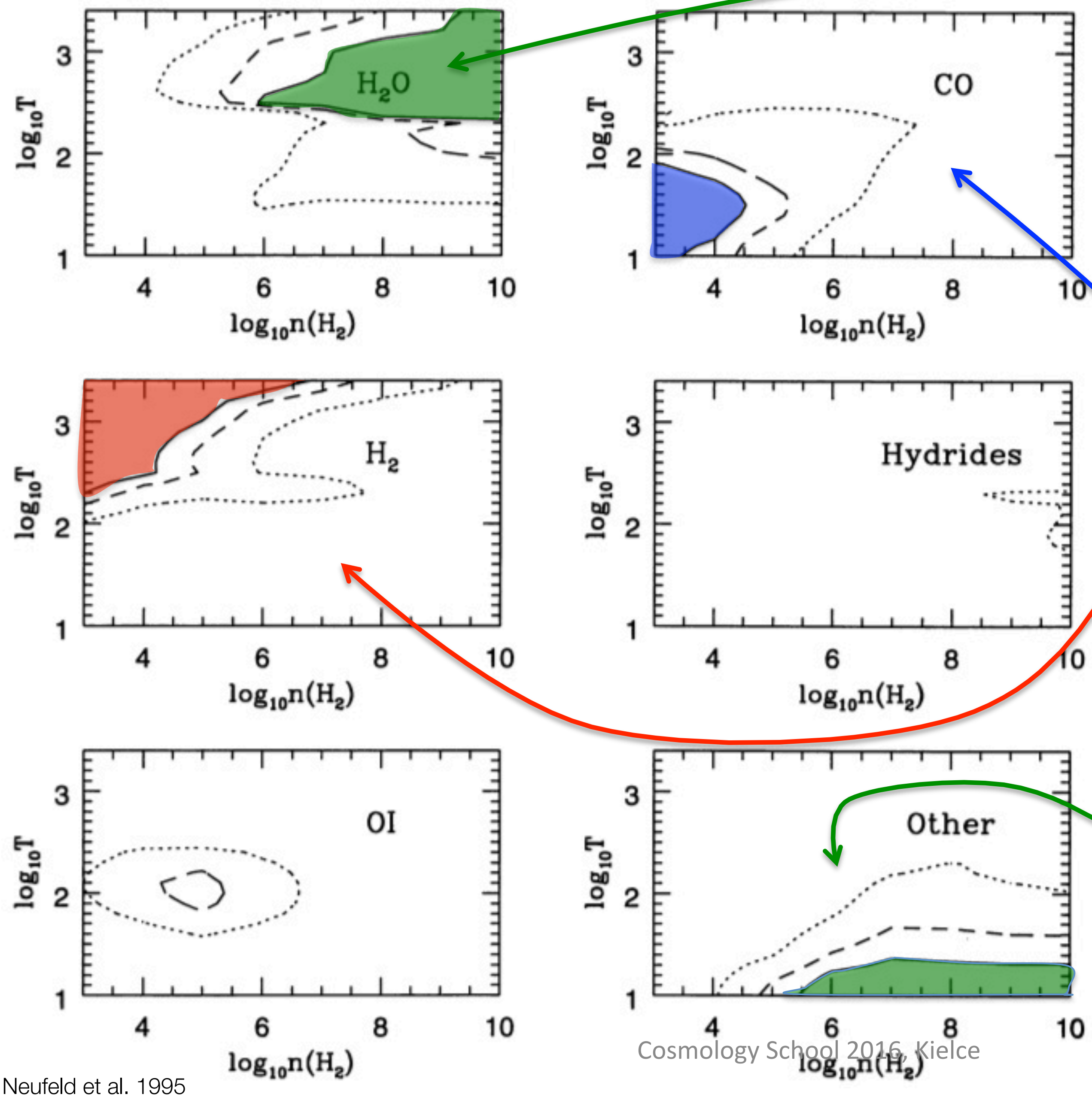
The figure shows the relative abundances of a molecule M, $x(M)=n(M)/n(H_2)$, for $n(H_2)=10^6$ cm^{-3}.

For T>500 K, all the oxygen not locked in CO, is in the form of water.

$$O + H_2 \rightarrow OH + H$$

$$OH + H_2 \rightarrow H_2O + H.$$

Contribution of coolants



Neufeld et al. 1995

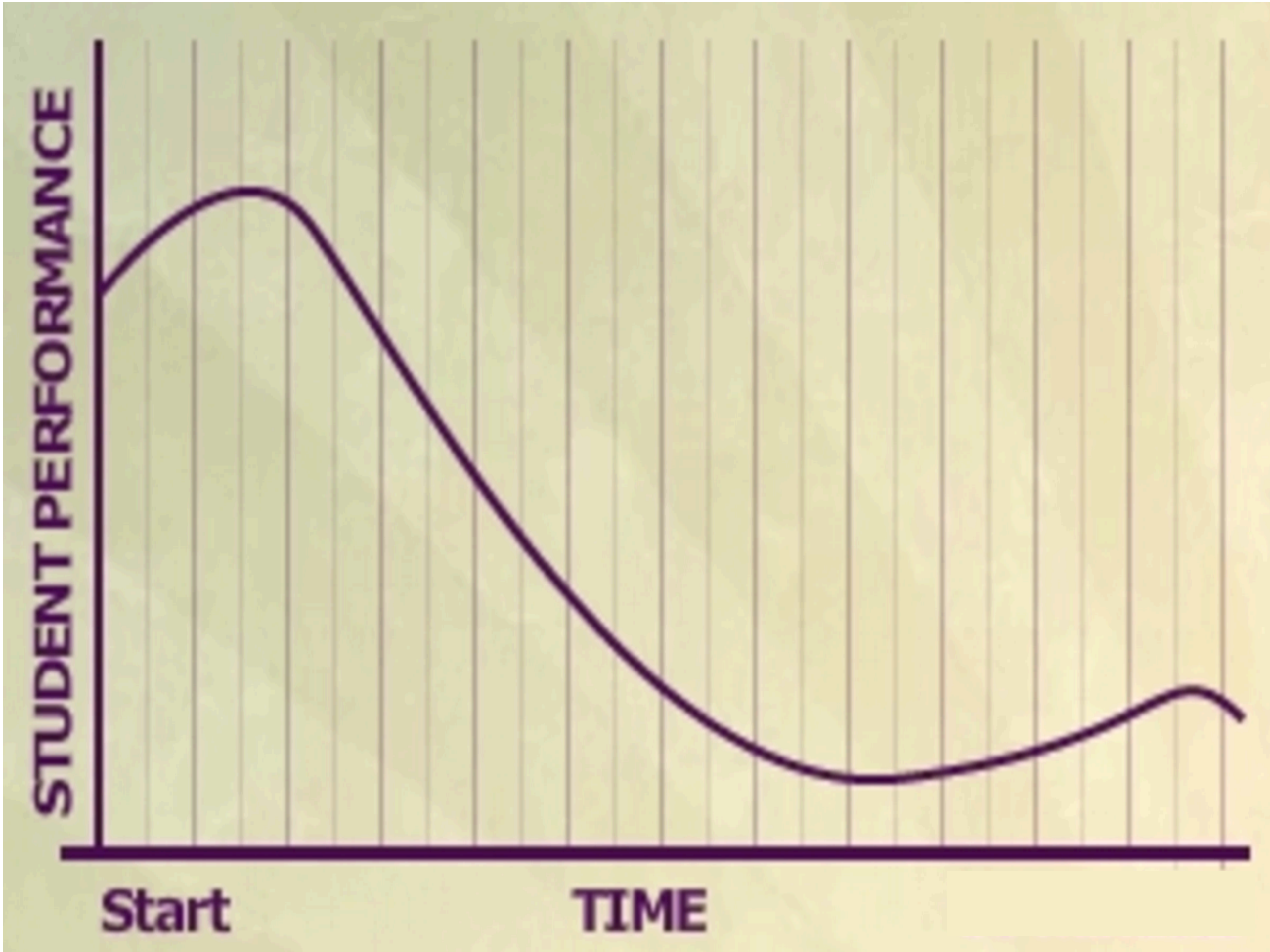
Fractions of the total cooling rate accounted for by emission of various coolants.

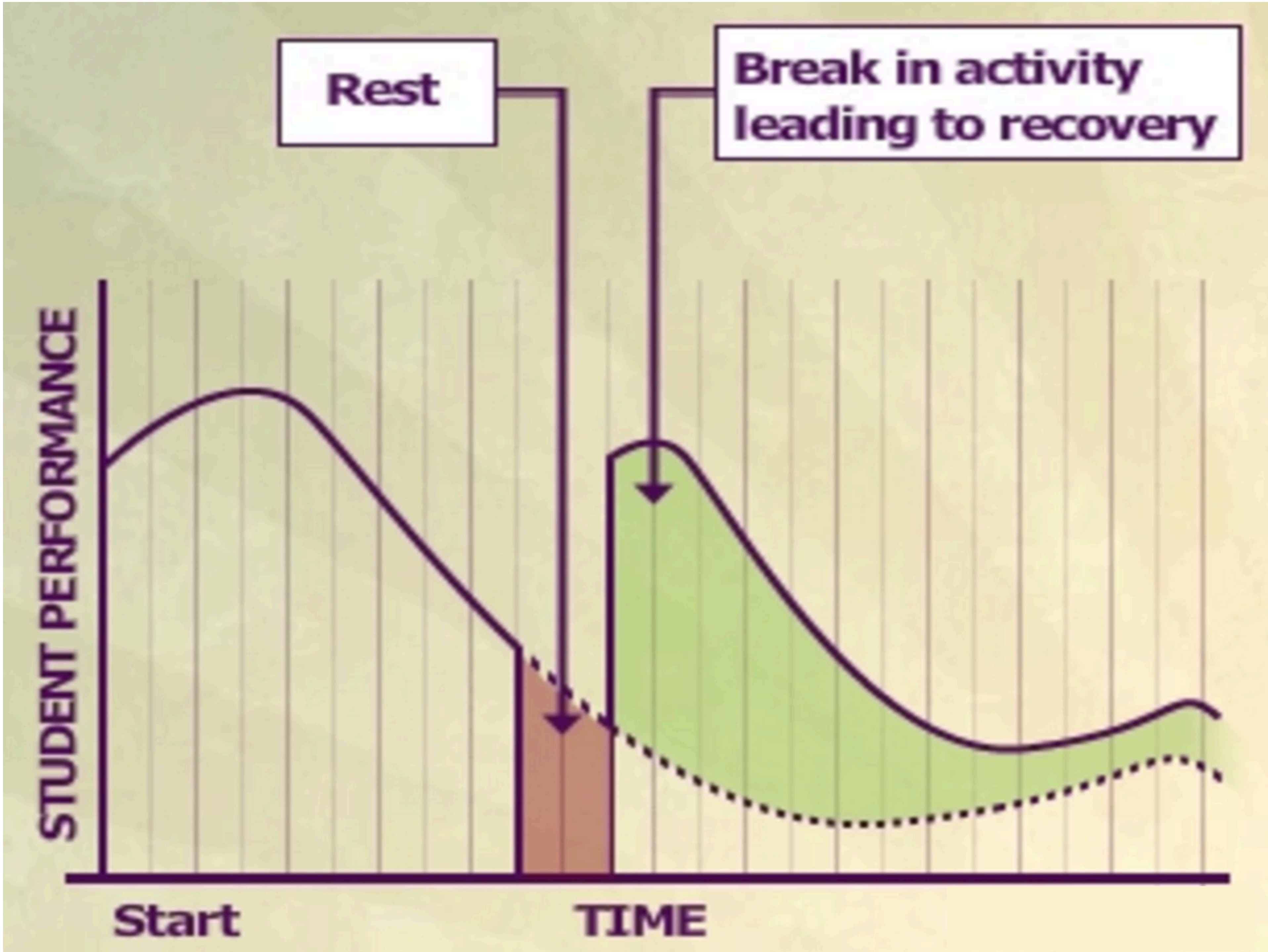
Contours correspond to
20% dotted
50% dashed
70% solid

Notes

- H (and He) cannot be collisionally excited at low T. It is there a poor coolant.
- At lower densities and temperatures, CO and O are dominant.
- At high densities, water (high T) and a lot of other molecules (low T) become dominant.

Numerical Simulations of Star Formation


STUDENT PERFORMANCE
Start
TIME



Rest
Break in activity leading to recovery
STUDENT PERFORMANCE
Start
TIME


Awake Now?

Numerical simulations

There is a large body of literature concerning the collapse of gas clouds. Various initial cloud shapes and initial density structures have been investigated. The effects of different EOS, radiation transport, and of magnetic fields have been studied. In the absence of the latter, these numerical simulations have shown that the key initial parameters are given by:

1) The initial thermal energy content: $\alpha = \dfrac{E_{therm}}{|E_{pot}|}$

2) The initial rotational energy content: $\beta = \dfrac{E_{rot}}{|E_{pot}|}$ $\qquad E_{rot} = \dfrac{1}{2} I \Omega^2$

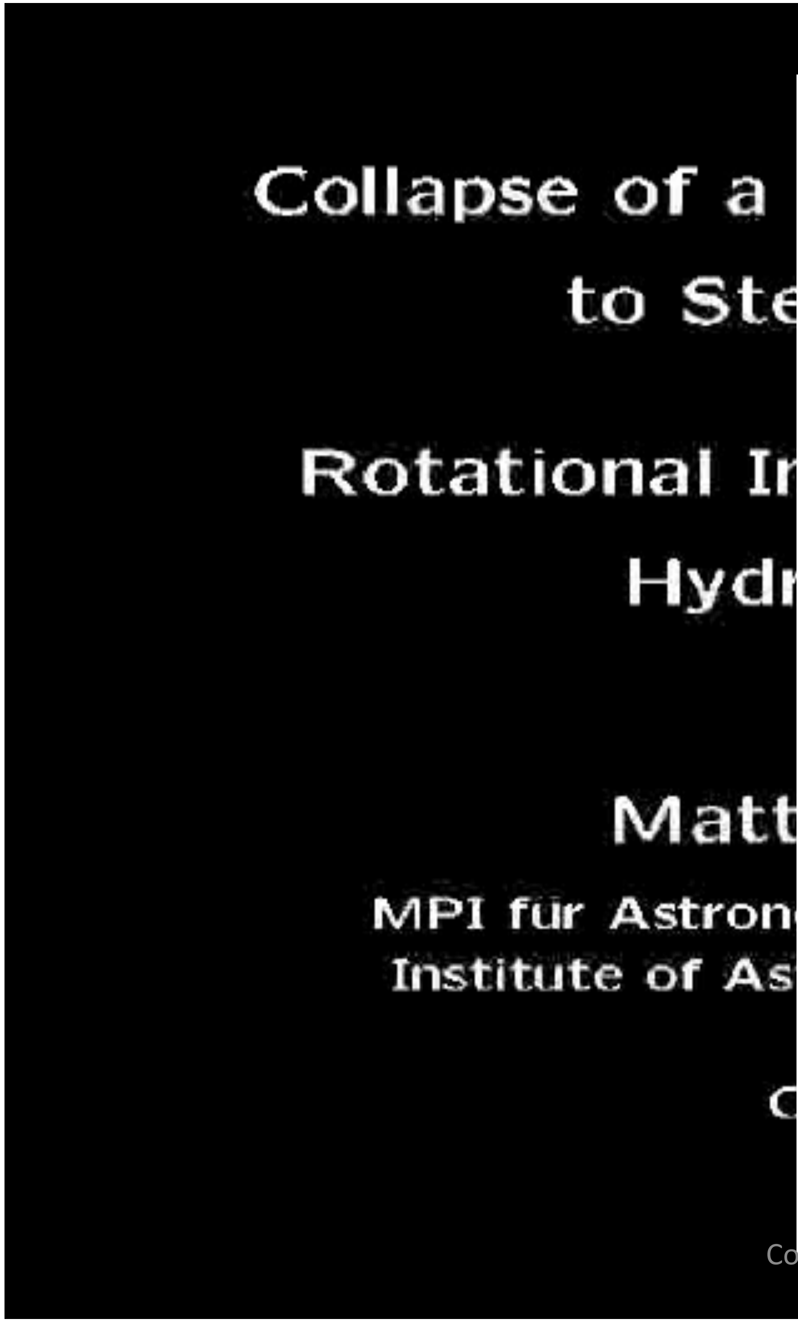
3) The exponent n of the initial power law density distribution $\rho \propto r^{-n}$

For $\beta > 0.274$, a spherical cloud is dynamically unstable to fragmentation.

Example 1: Bate 1998 : First 3D simulation. Single star formation with effect of rotation.

- Initial molecular cloud is rotating. First hydrostatic core rotating so fast that it is flattened.
- After a short time it goes from begin round to a bar-shaped object (dynamical instability).
- The ends of the bar rotate slower creating spiral arms. These create gravitational torques that transfer angular momentum from the centre of the object into the ends of the arms. The result is that some gas forms a large disc while, in the centre, the density and temperature increase rapidly.
- Molecular hydrogen dissociates at the centre and the second collapse to form the star occurs.
- The calculation stops just after the star forms.

Numerical Simulations II



- The Formation of Stars and Brown Dwarfs and the Truncation of Protoplanetary Discs in a Star Cluster (Bate, Bonnell, & Bromm)
- The calculation models the collapse and fragmentation of a molecular cloud with a mass 50 times that of our Sun. The cloud is initially 1.2 light-years (9.5 million million km) in diameter, with a temperature of 10 K.
- The cloud collapses under its own weight and very soon stars start to form.
- Surrounding some of these stars are swirling discs of gas which may go on later to form planetary systems like our own Solar System.
- The calculation took approximately 10^5 CPU hours running on up to 64 processors on the UKAFF supercomputer. In terms of arithmetic operations, the calculation required about 10^16 FLOPS (i.e. 10 million billion arithmetic ops).

Numerical simulations III

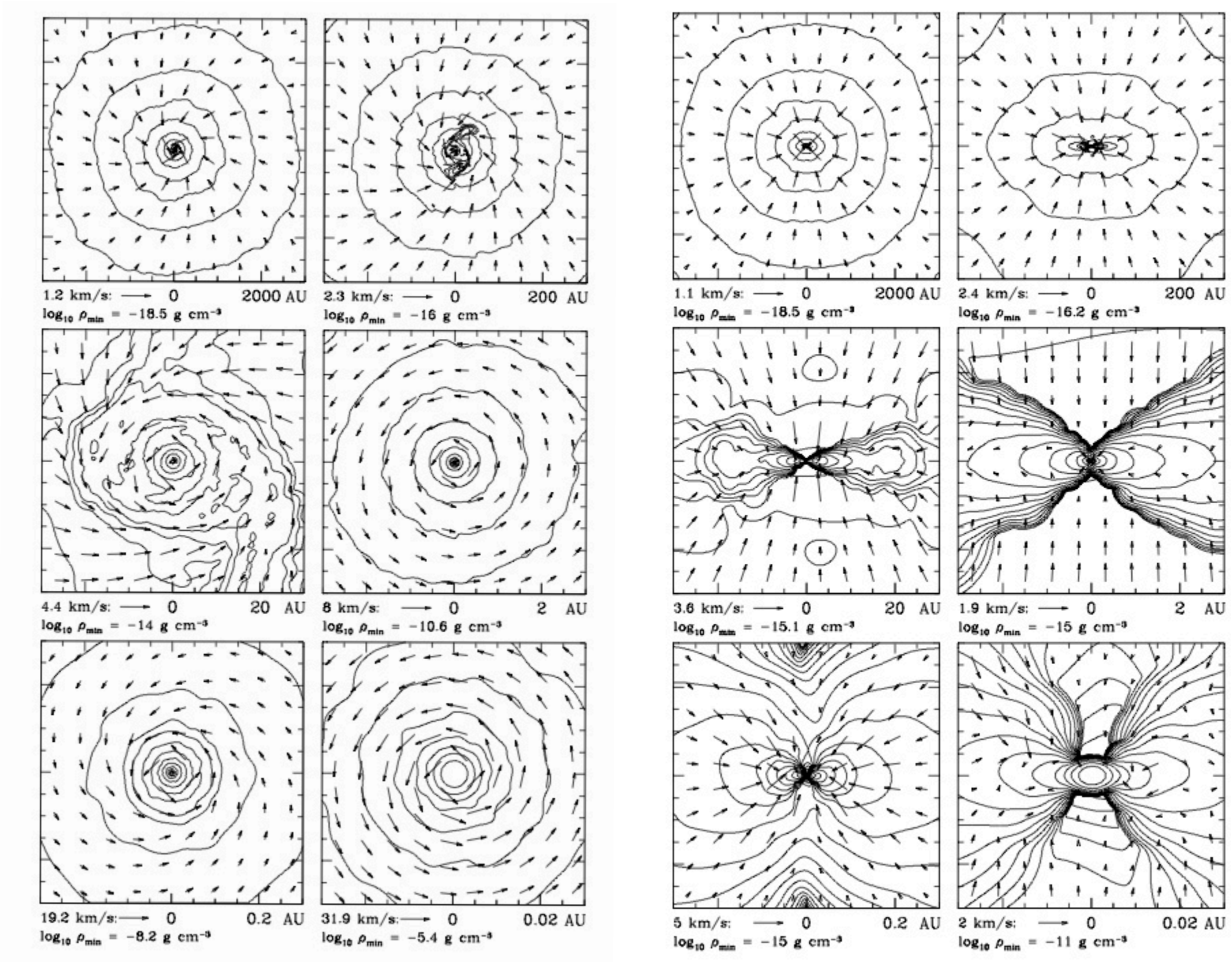


Fig. 3.— The state of the system at the end of the calculation. The six panels on the left give the density and velocity in the plane perpendicular to the rotation axis and through the stellar core. The six panels on the right give the density and velocity in a section down the rotation axis. In each case, the six consecutive panels give the structure on a spatial scale that is 10 times smaller than the previous panel to resolve structure from 3000 AU to $\approx 0.2\ R_\odot$. The remnant of the first hydrostatic core (now a disc with spiral structure), the inner circumstellar disc, and the stellar core are all clearly visible. The distance scale (in AU), velocity scale (in km s^{-1}), and logarithm of the minimum density (in g cm^{-3}) are given under each panel. The maximum density is 0.03 g cm^{-3} and the logarithm of density is plotted with contours every 0.5 dex.

Numerical simulations IV

Simulation outcome

- Isothermal collapse. First core forms with mass ≈ 0.01 $M_{\odot}$ and radius ≈ 7 AU. Rapidly rotating, oblate, and has $\beta \approx 0.34 > 0.274$, therefore dynamically unstable to the growth of non-axisymmetric perturbations
- At $t \approx 1.023\ t_{ff}$, after about 3 rotations, the first core becomes violently bar-unstable and forms trailing spiral arms. This leads to a rapid increase in maximum density as angular momentum is removed from the central regions of the first core (now a disc with spiral structure) by gravitational torques ($t > 1.015\ t_{ff}$).
- When the maximum temperature reaches 2000 K, molecular hydrogen begins to dissociate, resulting in a rapid second collapse to stellar densities ($t = 1.030\ t_{ff}$).
- The collapse is again halted at a density of ≈ 0.007 g cm^{-3} with the formation of the second hydrostatic, or stellar, core. The initial mass and radius of the stellar core are ≈ 0.0015 $M_{\odot}$) and ≈ 0.8 $R_{\odot}$, respectively.
- Finally, an inner circumstellar disc begins to form around the stellar object, within the region undergoing second collapse. The calculation is stopped when the stellar object has a mass of ≈ 0.004 $M_{\odot}$, the inner circumstellar disc has extended out to ≈ 0.1 AU, and the outer disc (the remnant of the first hydrostatic core) contains ≈ 0.08 $M_{\odot}$ and extends out to ≈ 60 AU.
- $\beta < 0.274$ in the region undergoing the second collapse: no formation of a close binary. Angular momentum removed by spiral arms (gravitational torques).

Numerical simulations V

Example 2:

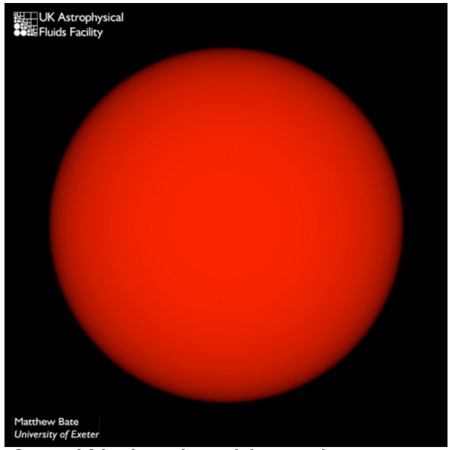
Bate 2009 : Hydrodynamic 3D simulation of a stellar cluster

Models the collapse and fragmentation of a 500 solar mass cloud. The calculation produces a cluster containing more than 1250 stars and brown dwarfs to allow comparison with clusters such as the Orion Trapezium Cluster.

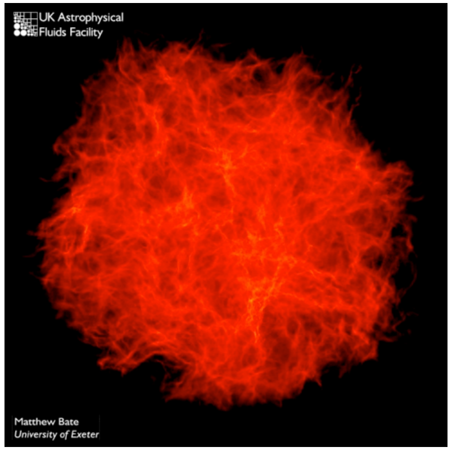
Parameter	Initial condition
R_0	0.8 pc
M_0	500 M_{sun}
v	supersonic turbulent
T	10 K
μ	2.46
T_{ff}	190 000 yrs
Stop	285 000 yrs
Radiation	not included
Magnetic fields	not included
Numerical method	SPH
Stars and BD	sink particles
Accretion radius	5 AU
Min. binary sep.	1 AU
EOS	piecewise polytropic
N particles	35 Mio

Numerical simulations VII

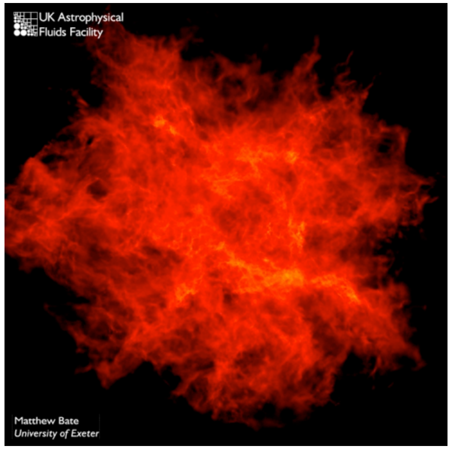
Bate 2009



0 yr: We begin with such a gas cloud, 2.6 light-years across, and containing 500 times the mass of the Sun. The images measure 1 pc (3.2 lightyears across).



38,000 yr: Clouds of interstellar gas are seen to be very turbulent with supersonic motions.

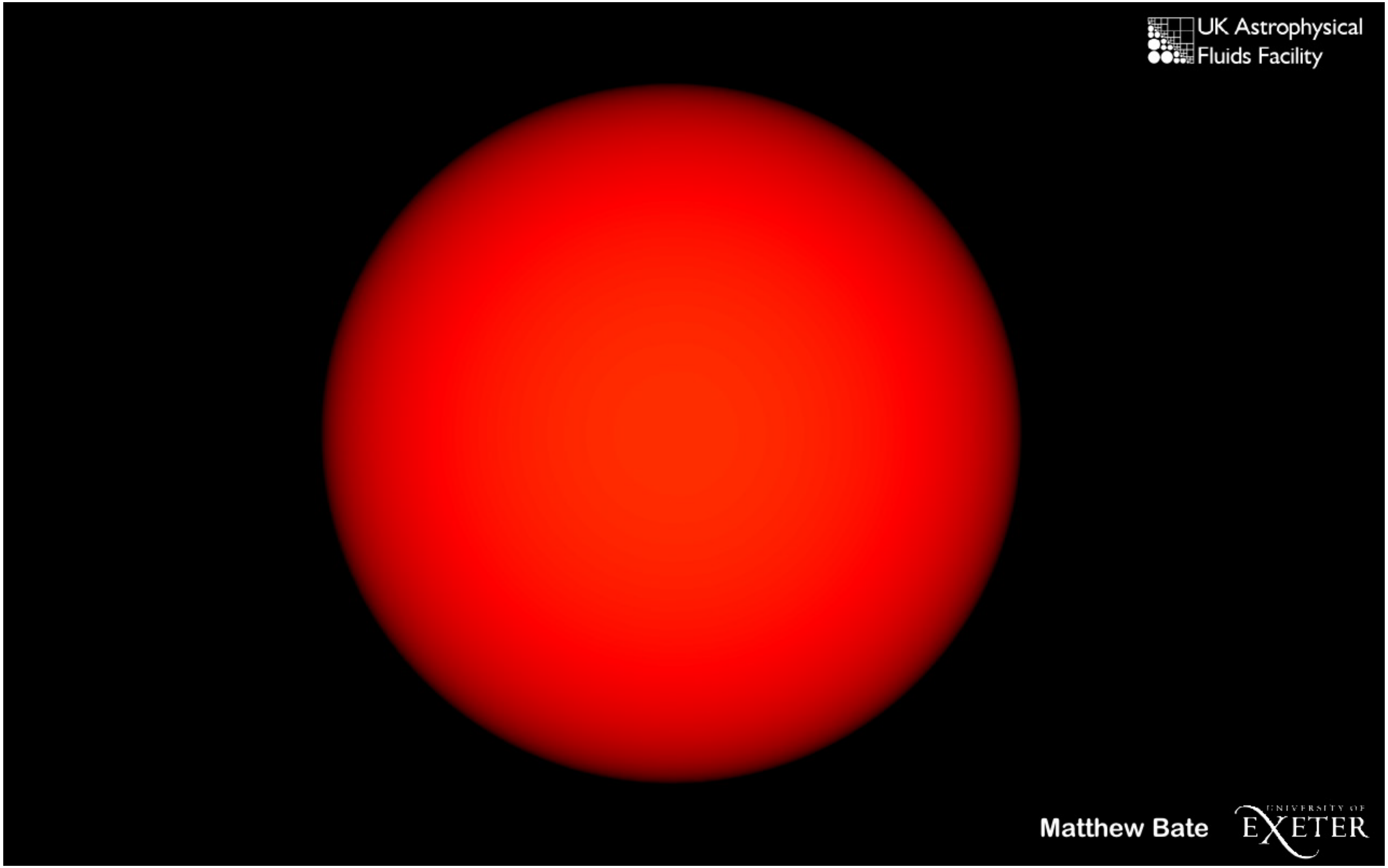


76,000 yr: As the calculation proceeds, the turbulent motions in the cloud form shock waves that slowly damp the supersonic motions.

152,000 yr: When enough energy has been lost in some regions of the simulation, gravity can pull the gas together to form dense "cores".

190,000 yr: The formation of stars and brown dwarfs begins in the dense cores. As the stars and brown dwarfs interact with each other, many are ejected from the cloud.

The cloud and star cluster at the end of simulation (which covers 210,000 years so far). Some stars and brown dwarfs have been ejected to large distances from the regions of dense gas in which the star formation occurs.


UK Astrophysical
Fluids Facility
Matthew Bate
UNIVERSITY OF
EXETER

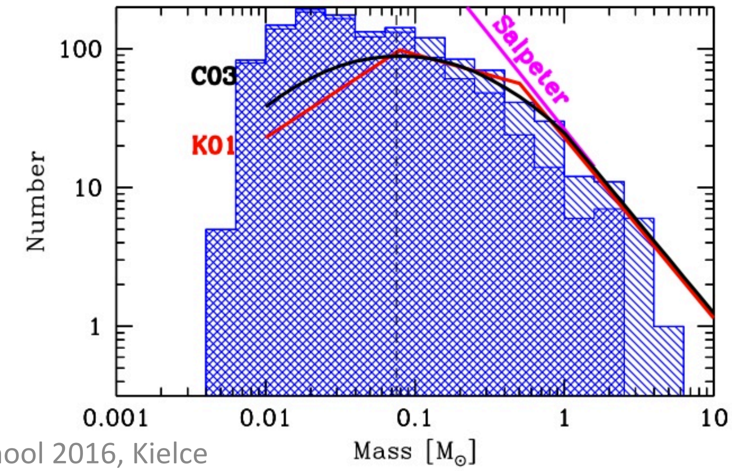
Numerical simulations VIII

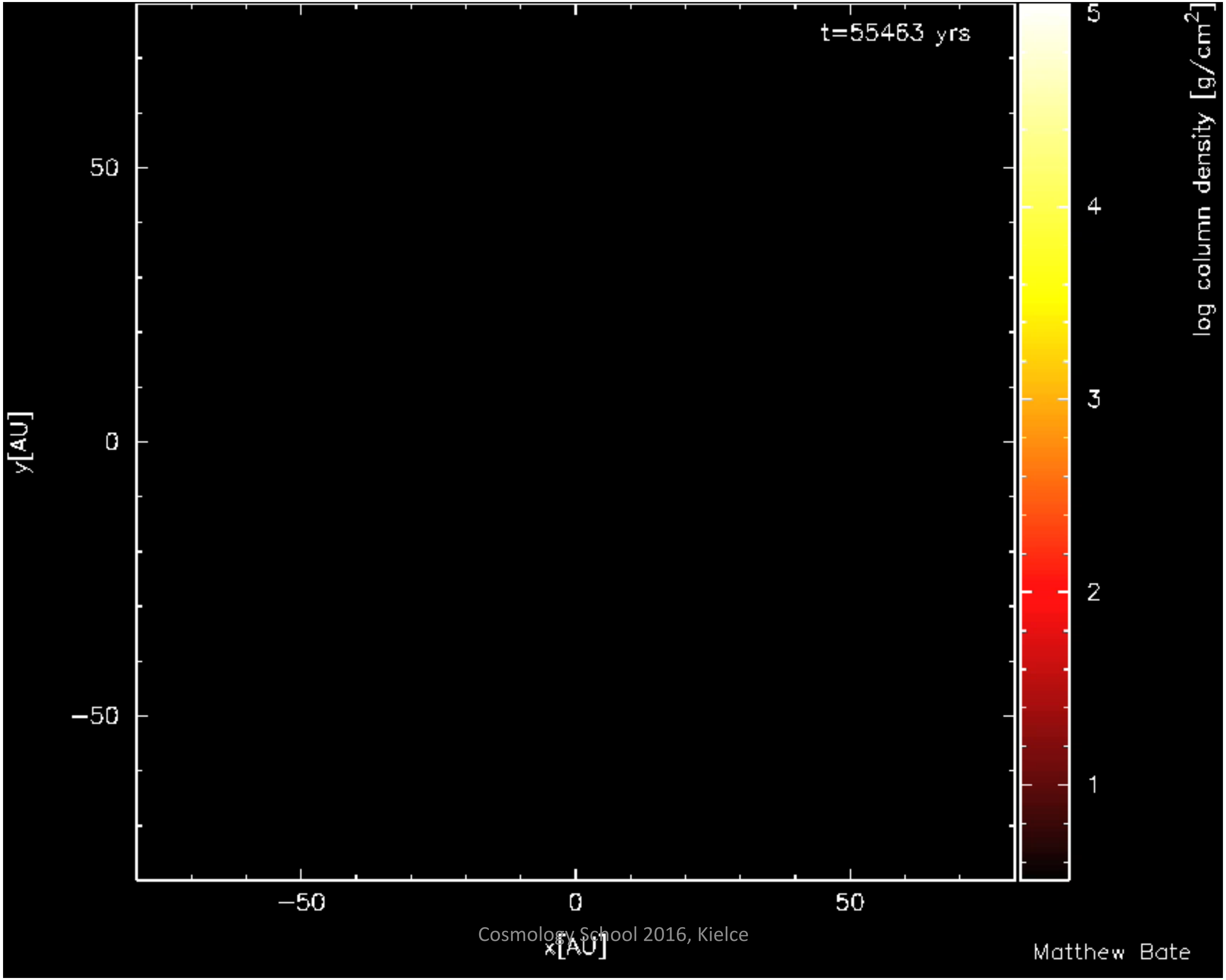
Main simulation results

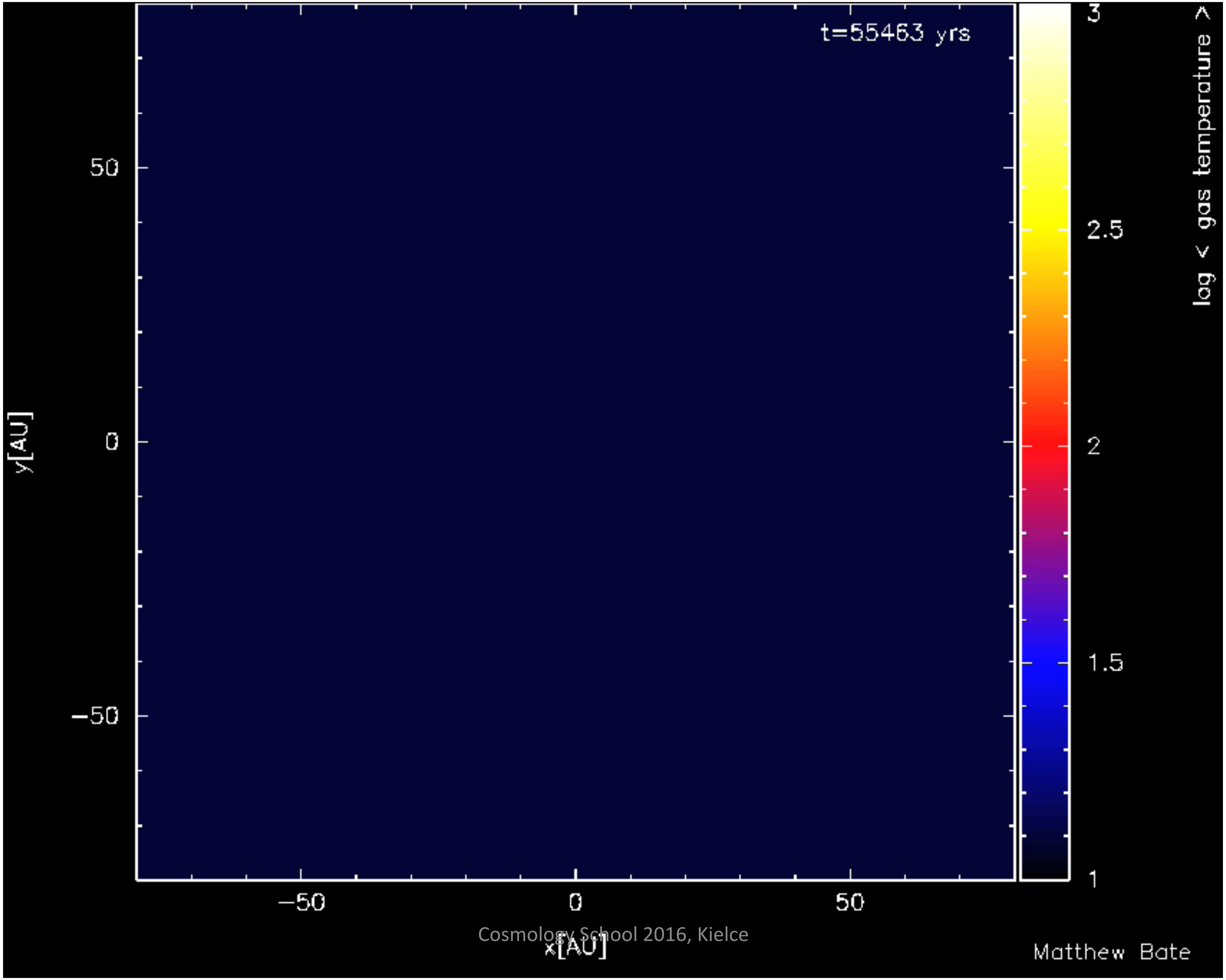
1) Since all sink particles (and thus stars/BD) are created from pressure-supported fragments, their initial masses are just a few Jupiter masses, as given by the opacity limit for fragmentation. Subsequently, they may accrete large amounts of material to become higher-mass brown dwarfs (< 75 $M_{Jupiter}$) or stars (>75 $M_{Jupiter}$), but all the stars and brown dwarfs begin as these low-mass pressure-supported fragments.

2) The IMF originates from competition between accretion and ejection which terminates the accretion and sets an object's final mass. Stars and brown dwarfs form the same way, with similar accretion rates from the molecular cloud, but stars accrete for longer than brown dwarfs before undergoing the dynamical interactions that terminate their accretion.

3) The calculations produce an IMF with a similar form to the observed IMF, including a Salpeter-type slope at the high-mass end but they over-produce brown dwarfs. It is likely due to the absence of radiative feedback and/or magnetic fields in the calculation.




t=55463 yrs
50
0
−50
−50
0
50
x[AU]
y[AU]
5
4
3
2
1
log column density [g/cm^2]
Matthew Bate


t=55463 yrs
y[AU]
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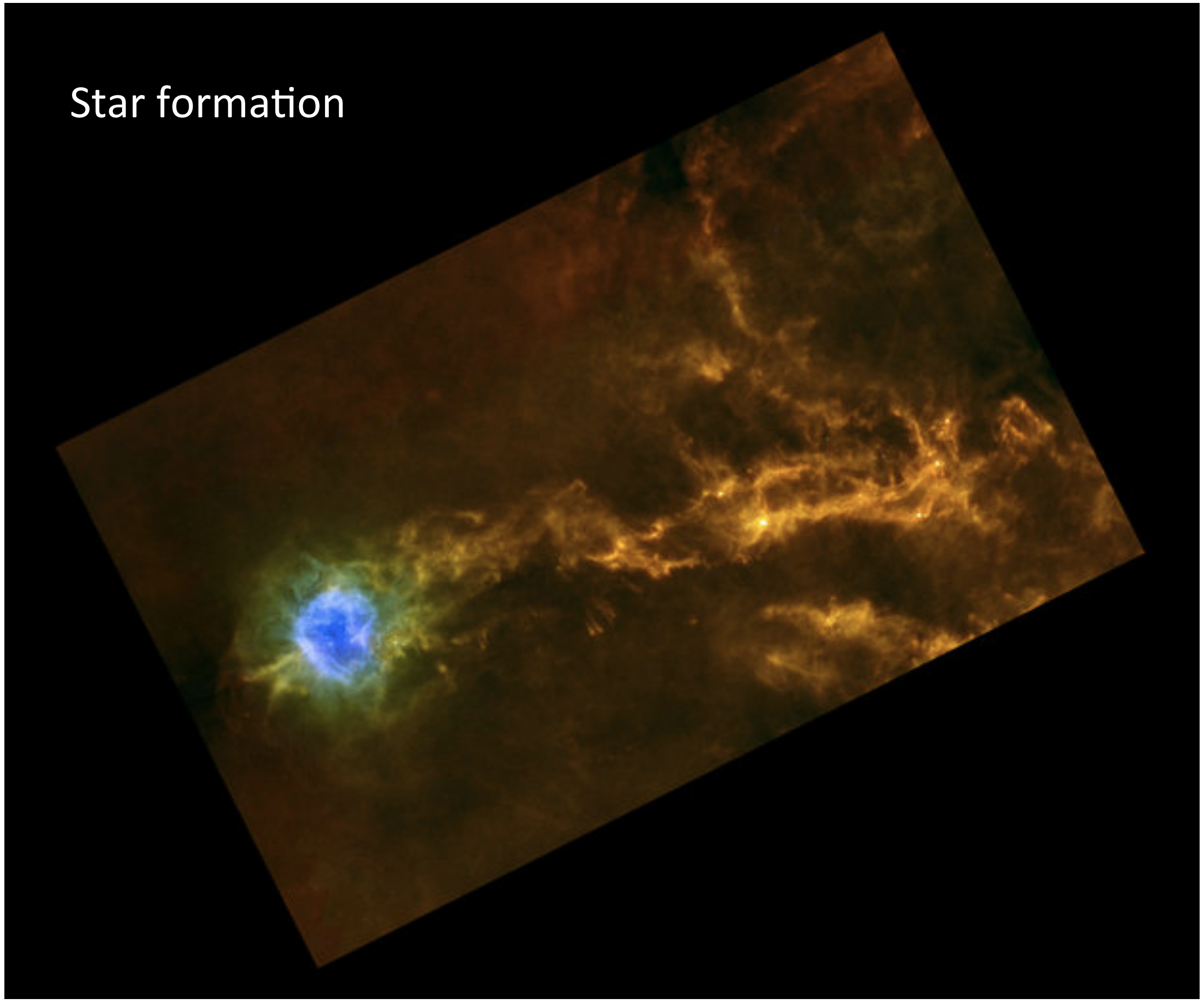
log < gas temperature >
Matthew Bate



• ESA's Herschel space observatory has revealed that nearby interstellar clouds contain networks of tangled gaseous filaments.
• The filaments are huge, stretching for tens of light years through space and Herschel has shown that newly-born stars are often found in the densest parts of them.
• One filament imaged by Herschel in the Aquila region contains a cluster of about 100 infant stars.
•Herschel has shown that, regardless of the length or density of a filament, the width is always roughly the same.

The Formation of the First Stars

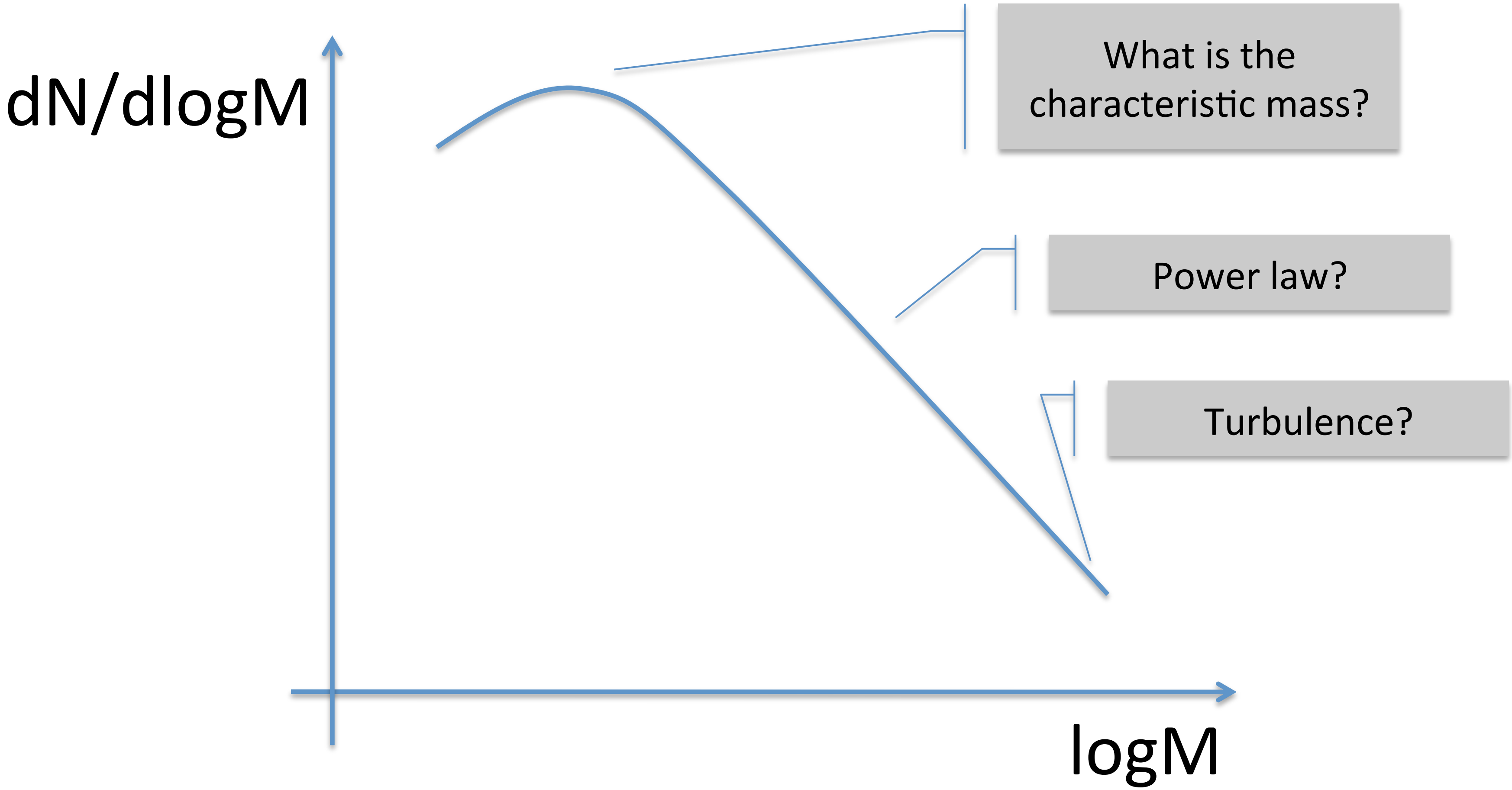
Simplified physics for the first stars

- No magnetic field is present yet (?)
- Since only primordial fusion : only H/D/He
  -> no metals -> no dust
- Initial physical conditions are assumed to follow ΛCDM (Cold Dark Matter)

$\Rightarrow$ First Stars

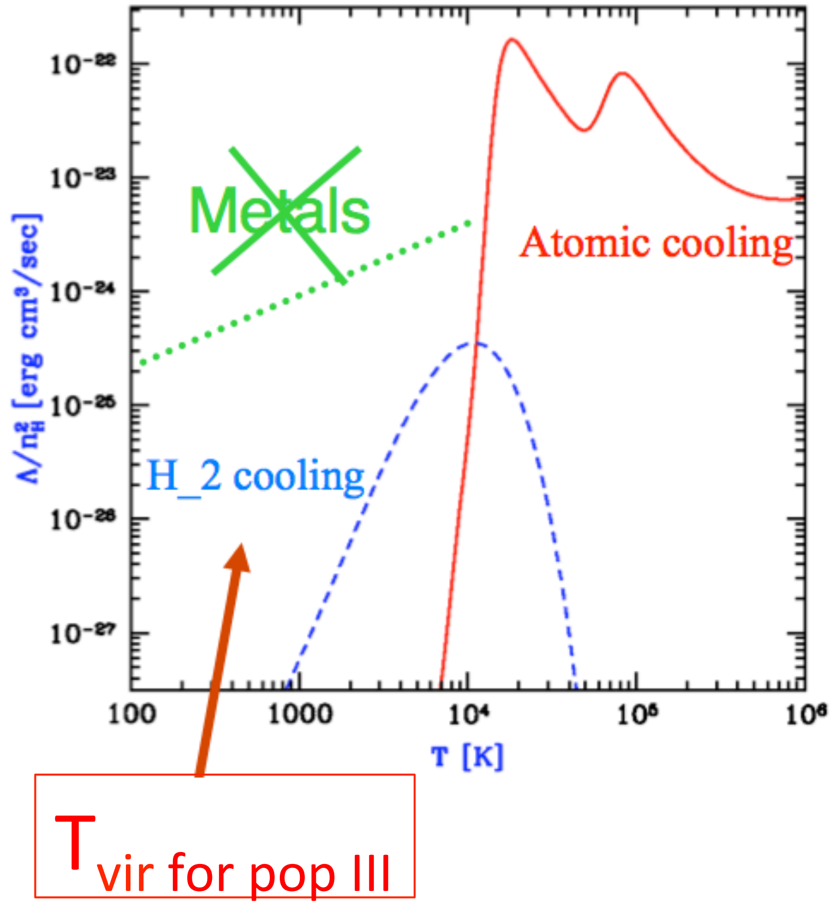
= CDM + atomic & molecular φsics of H/D/He

Other important assumption : the IMF for pop III stars …

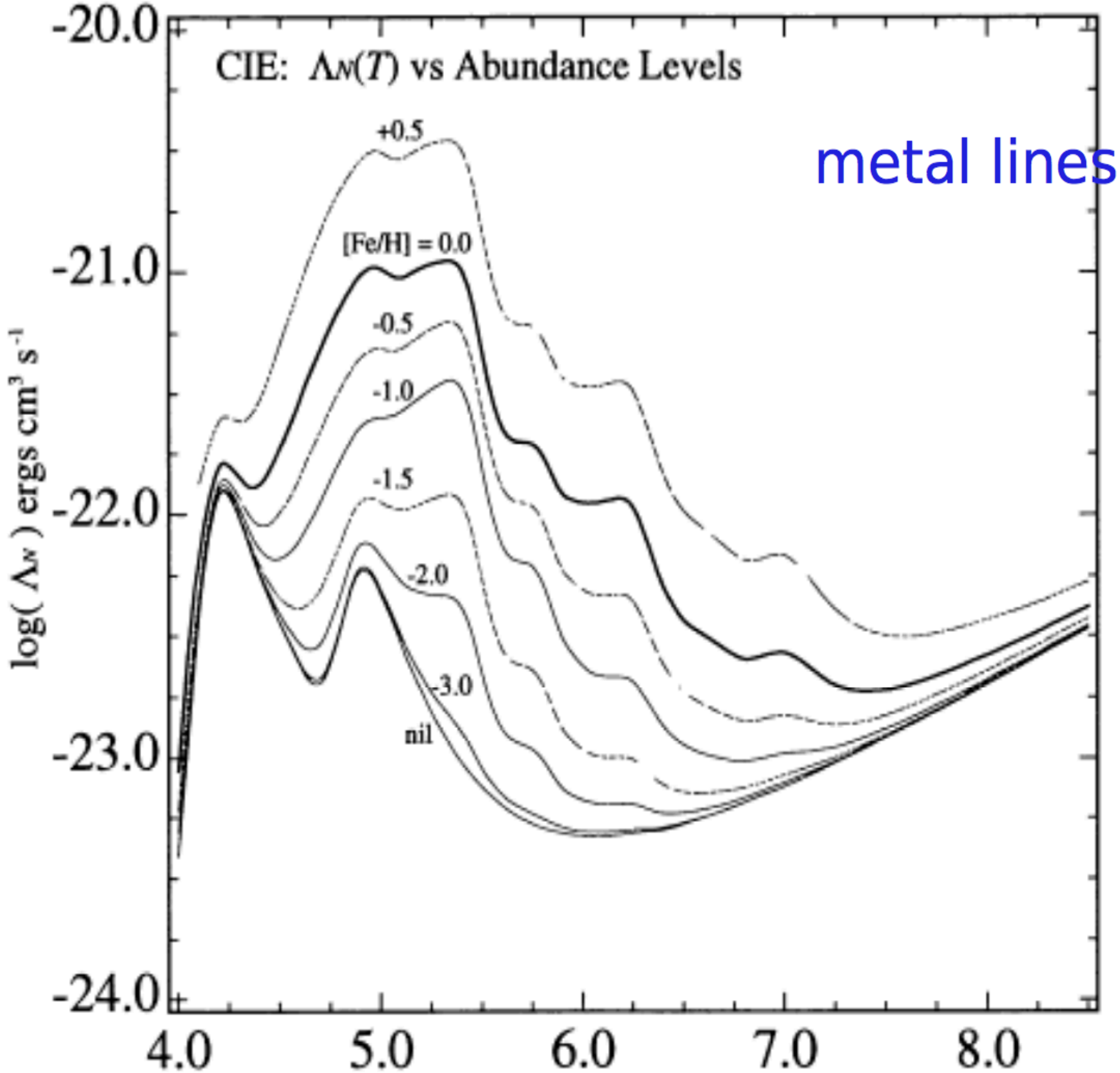


How could pop III stars cool down?

- Cooling rate of primordial gas as a function of temperature
- Shown is the contribution from atomic hydrogen and helium (solid line), as well as that from molecular hydrogen (dashed line).
- Atomic hydrogen line cooling is very efficient at temperatures of T > 10^4 K, whereas at lower temperatures, cooling has to rely on H_2, which is a poor coolant.
- This is the regime of the minihalos (~ $10^6 M_\odot$), hosting the formation of the first stars.



SUTHERLAND & DOPITA (1993, ApJS, 88, 253)

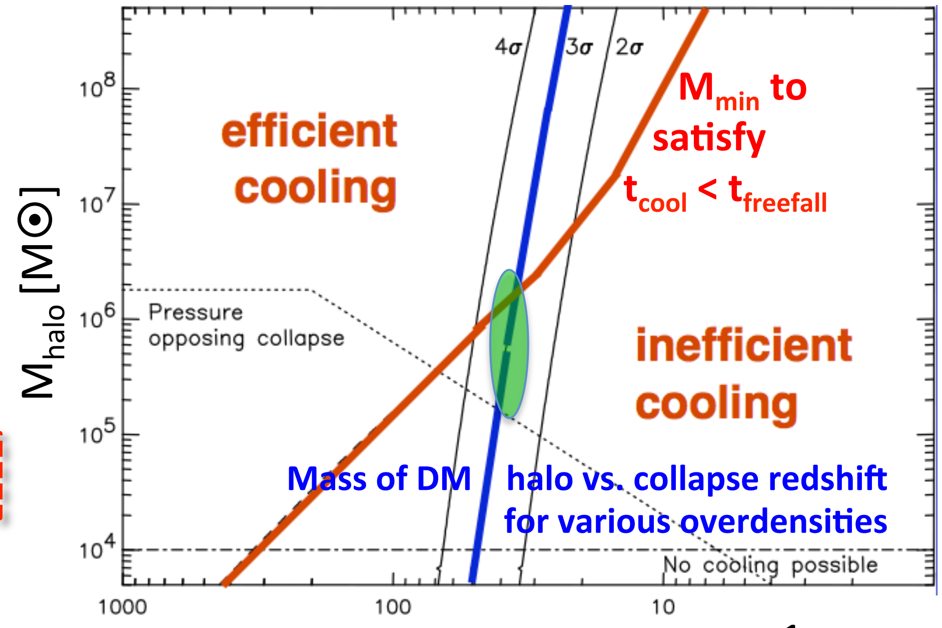


metal lines dominate cooling

Log T (K)

Regions of primordial star formation

- Gravitational Evolution of CDM
- Gas microφsics (H_2 cooling):
  - Can gas sufficiently cool?
  - $t_{cooling} < t_{freefall}$
- Collapse of First Luminous Objects expected: Mini-halos
  - At $z_{collapse} = 20 – 30$
  - $M_{total} \sim 10^6 M_\odot$



Bromm (2013, 1305.5178v2)

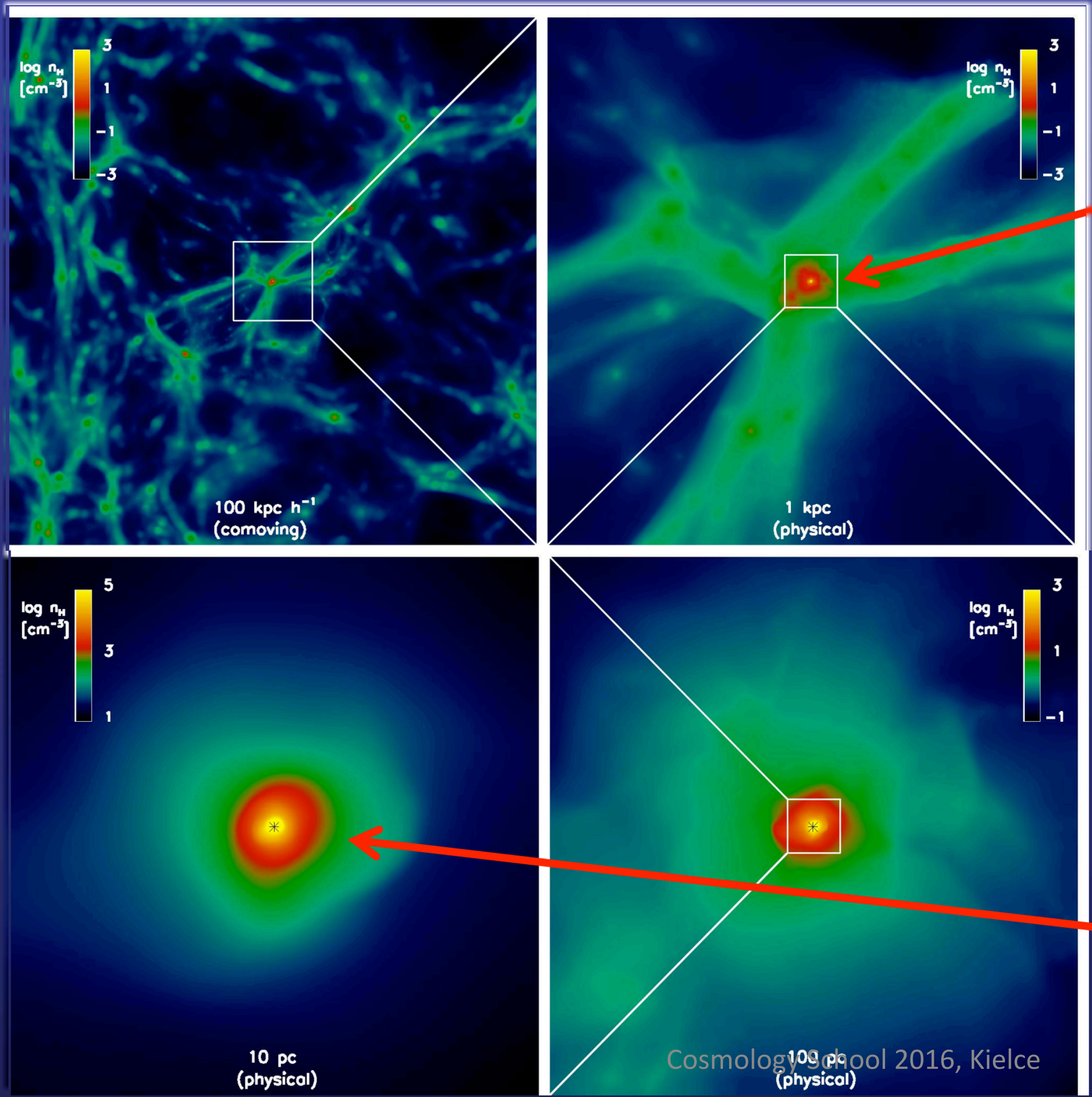
• A simple, but intuitively appealing and useful, answer is provided by the Rees-Ostriker-Silk criterion, that the cooling time has to be shorter than the free-fall time: $t_{cool} < t_{ff}$.

• If this criterion is fulfilled, a gas cloud will be able to condense to high densities, and possibly undergo gravitational collapse.

• These important timescales are defined as follows: $t_{ff} \simeq 1/\sqrt{G\rho}$ and $t_{cool} \simeq nk_BT/\Lambda$ where Λ is the cooling function.

Formation of a Population III Star

(Stacy, Greif & Bromm 2010, MNRAS, 403, 45)



Minihalo:

$M \sim 10^6 M_o$

$R \sim 100$ pc

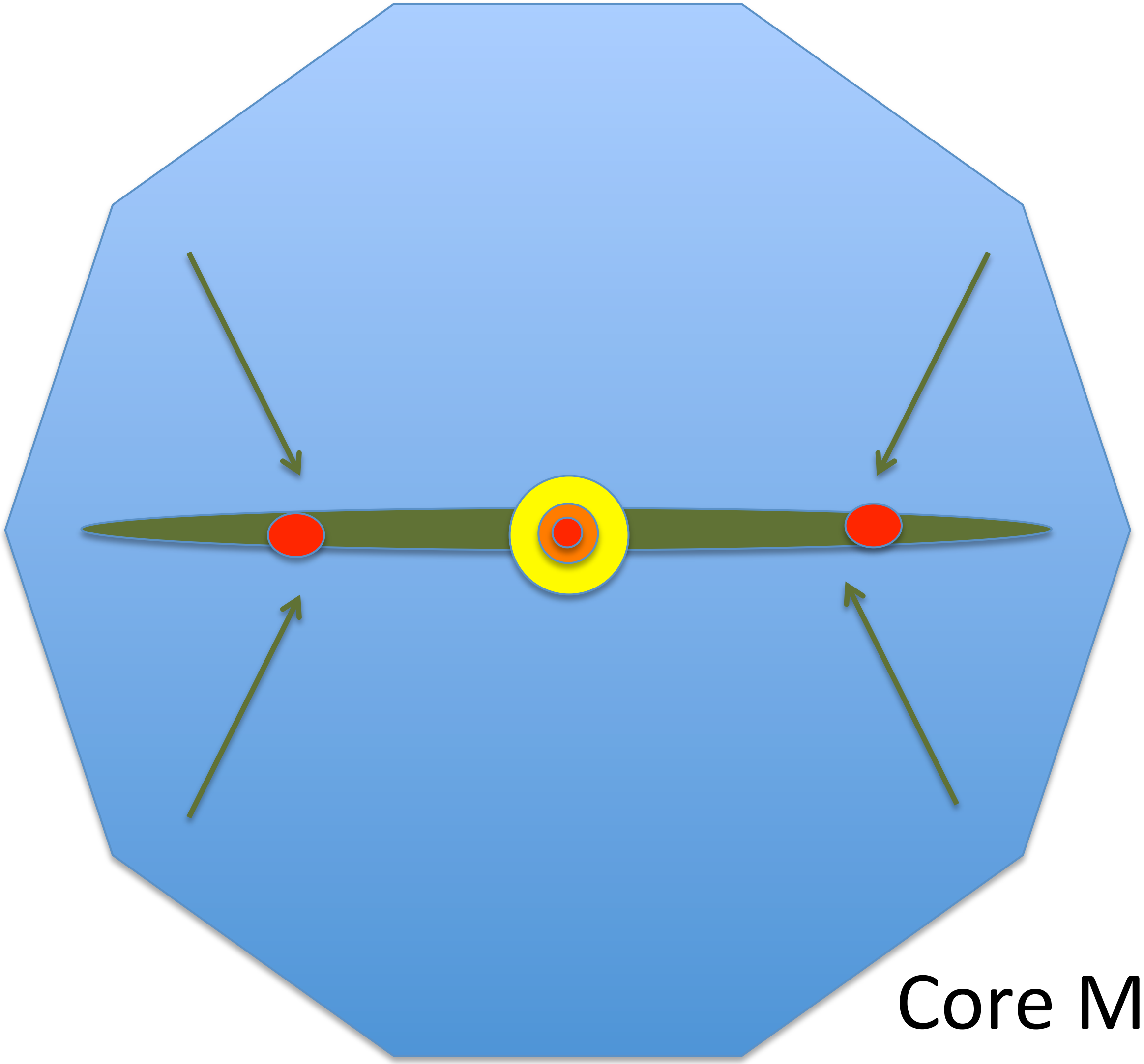
$z \sim 20$

Clump:

$M \sim 10^3 M_o$

$R \sim 1$ pc

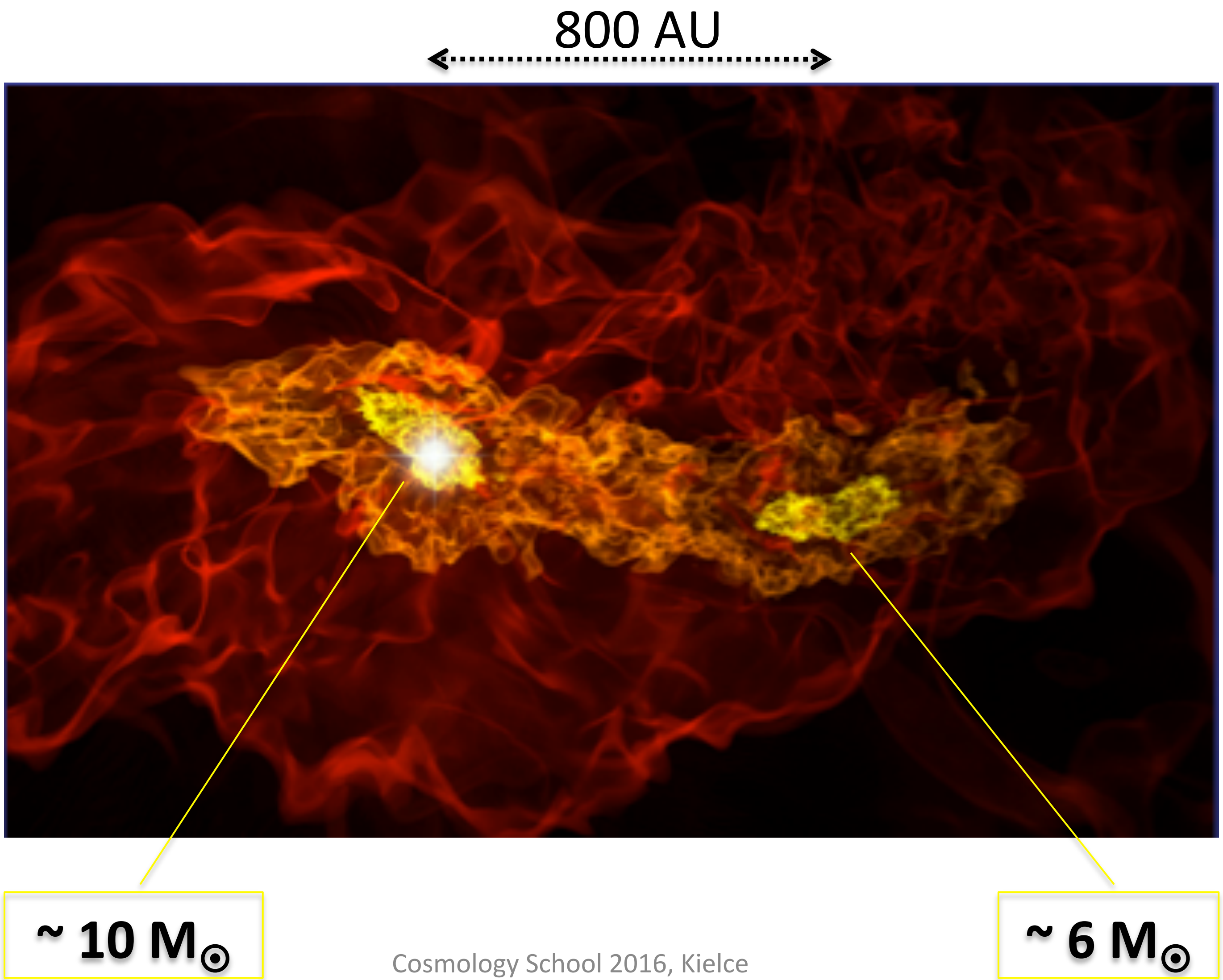
Schematic view of the primordial star formation: qualitatively similar to Today



Core Mass ~ M_J

Early POP III binaries

(Turk, Abel & O'Shea 2009, Science, 325, 601)



Until recently, it was assumed that pop III cores formed just a single star, because no fragmentation was seen in the simulations during the formation of the first protostar. As a result, attempts to estimate the final mass of the primordial stars have concentrated on balancing the inward accretion of gas from the collapsing core by the radiative feedback from the young protostar, with various calculations predicting a final mass in the range 30–300 Msun.

The fragmentation of the gas arises from the chaotic turbulent flows that feed the inner regions of the star-forming minihalos.

When replacing very high density collapsing regions with accreting 'sink' particles each of which represents an individual protostar, smaller stars are created.

Early POP III binaries

(Turk, Abel & O'Shea 2009, Science, 325, 601)

« Previous high resolution cosmological simulations predict the first stars to appear in the early universe to be very massive and to form in isolation. Here we discuss a cosmological simulation in which the central 50MSun clump breaks up into two cores, having a mass ratio of two to one. »

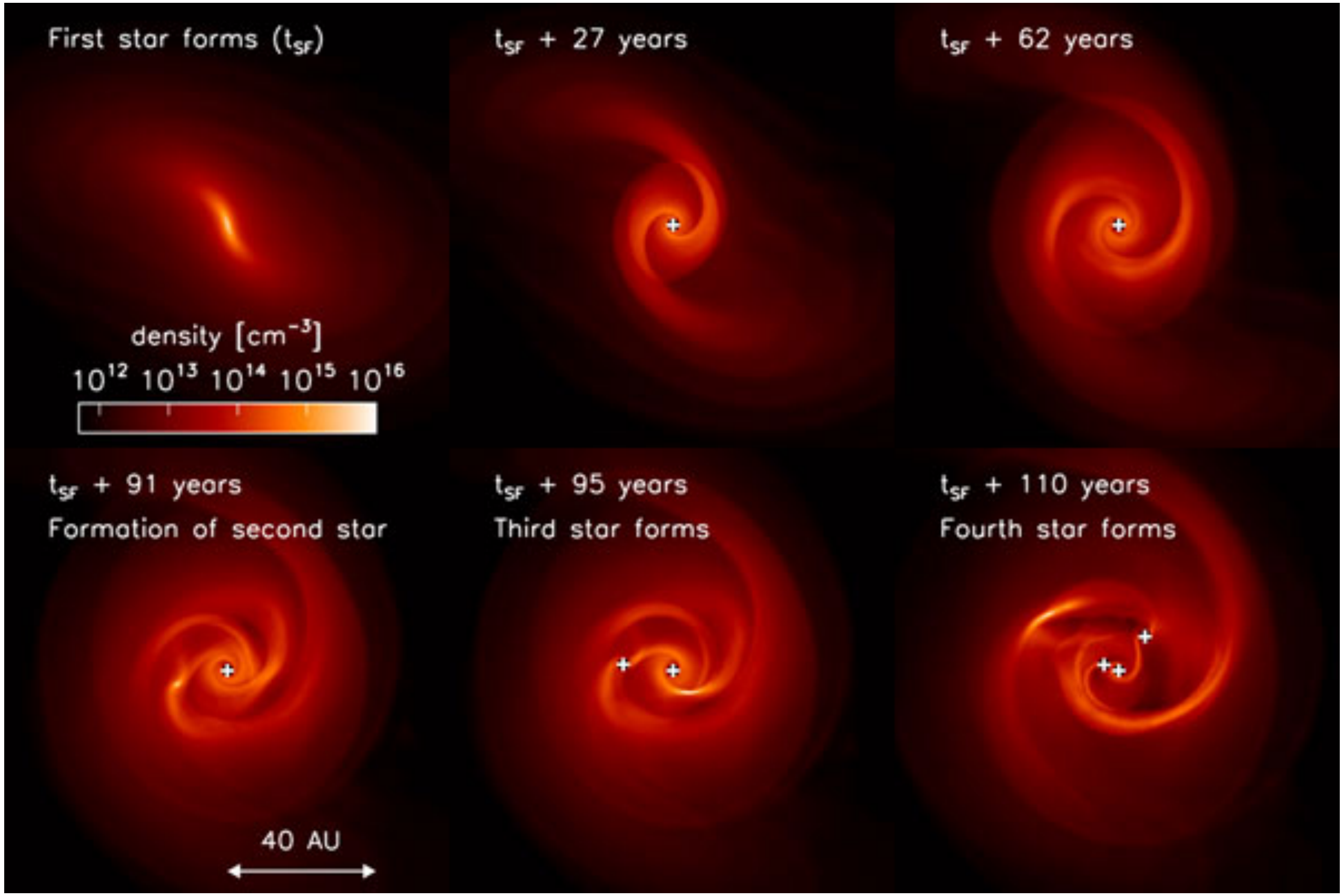
« The problem of "finding" fragmentation in cosmological halos may be one of poor sampling; if fragmentation is rare, the small number of published calculations likely will not sample those halos in which it could occur. In particular, if fragmentation is more likely to occur in halos that undergo rapid merger history, the current means of ab initio simulation of Population III star formation may be ill-equipped to study its likelihood and relevance. »

Pop III Star Formation: disk fragmentation (Clark et al. 2010)

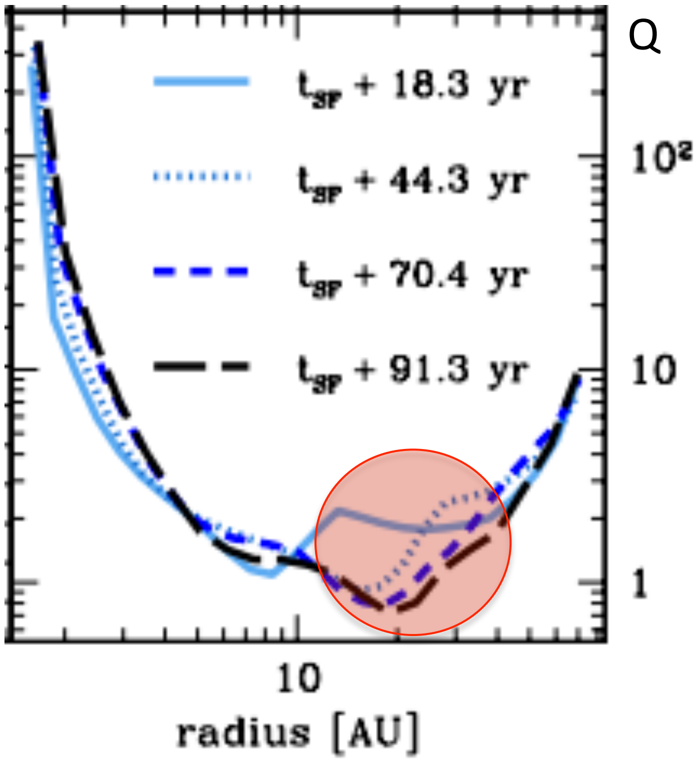
• Because gas in the early universe did not contain metals, it was thought that primordial stars were solitary massive objects.

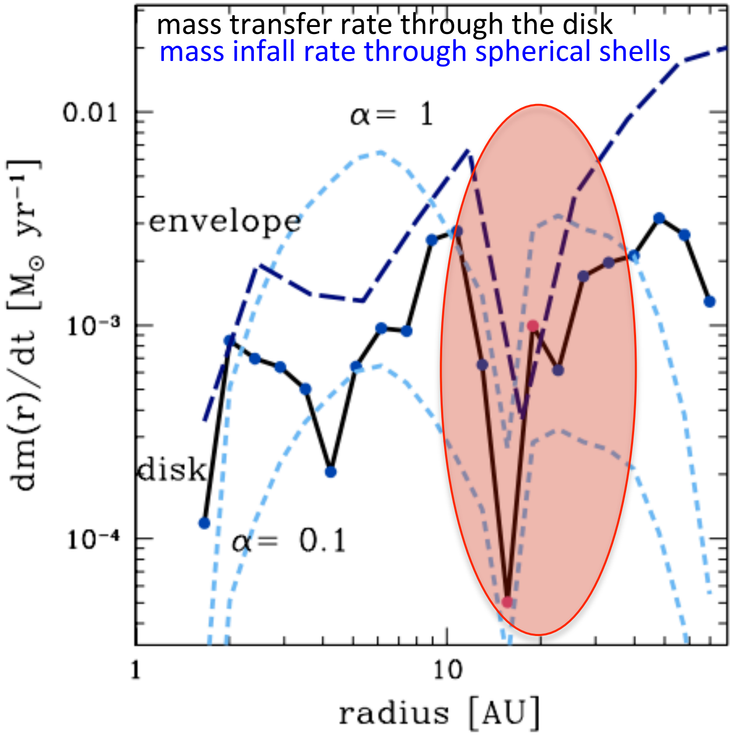
• But, the accretion disks that formed around the first stars were found to be highly susceptible to fragmentation.

• Therefore, instead of forming in isolation, the first stars almost always occur as members of multiple stellar systems, with separations as small as the distance between Earth an the Sun.



Pop III Star Formation: Disk Fragmentation (Clark et al. 2011)

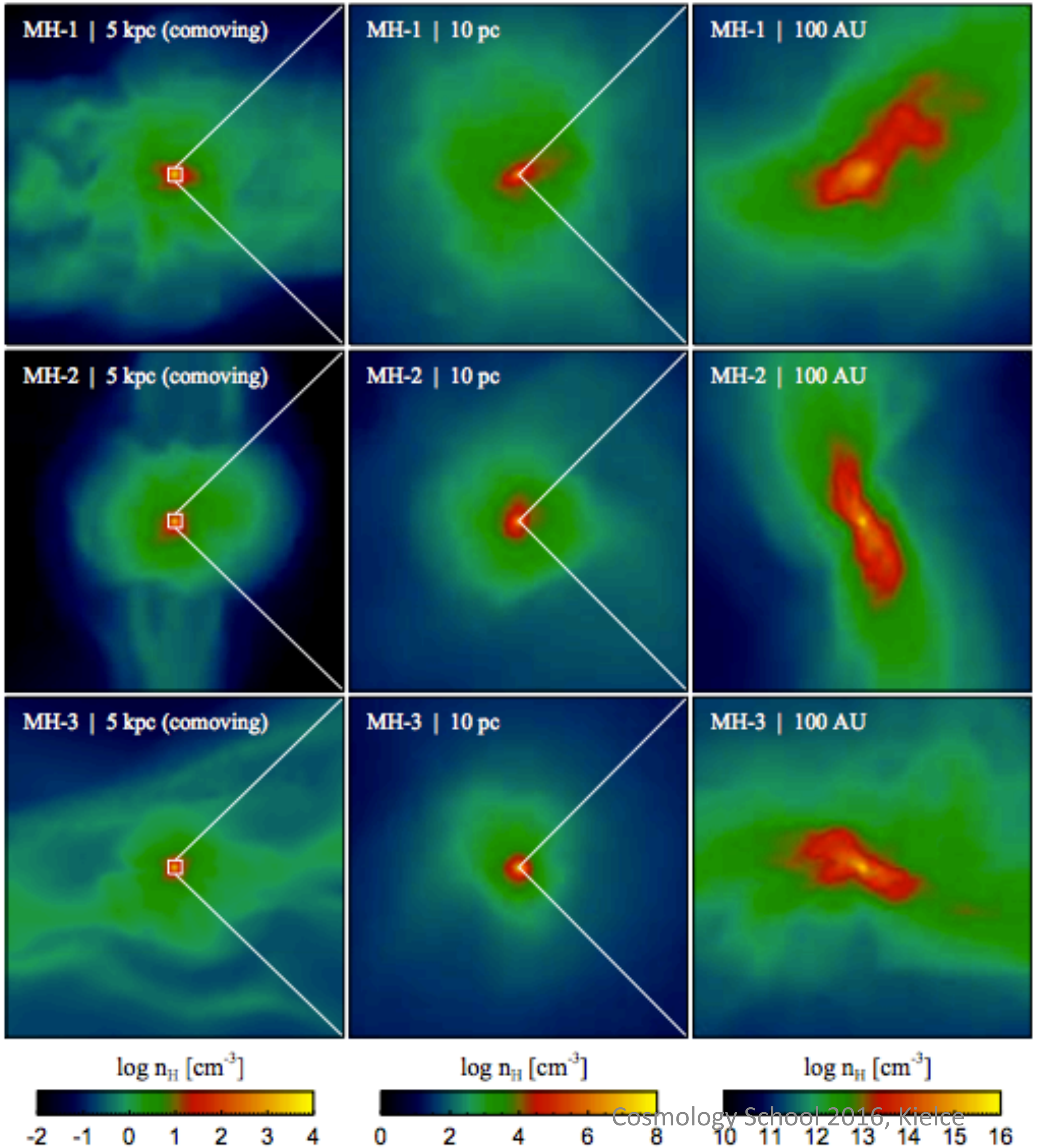




Left: the Toomre 'Q' parameter provides a measure of the gravitational instability of the disk. For high Q, the disk is stable, while for values around 1, the disk is unstable to fragmentation. As the disk grows, the value of Q remains around 1 in the outer regions, and so the dynamics of the disk is dominated by gravitational instabilities.

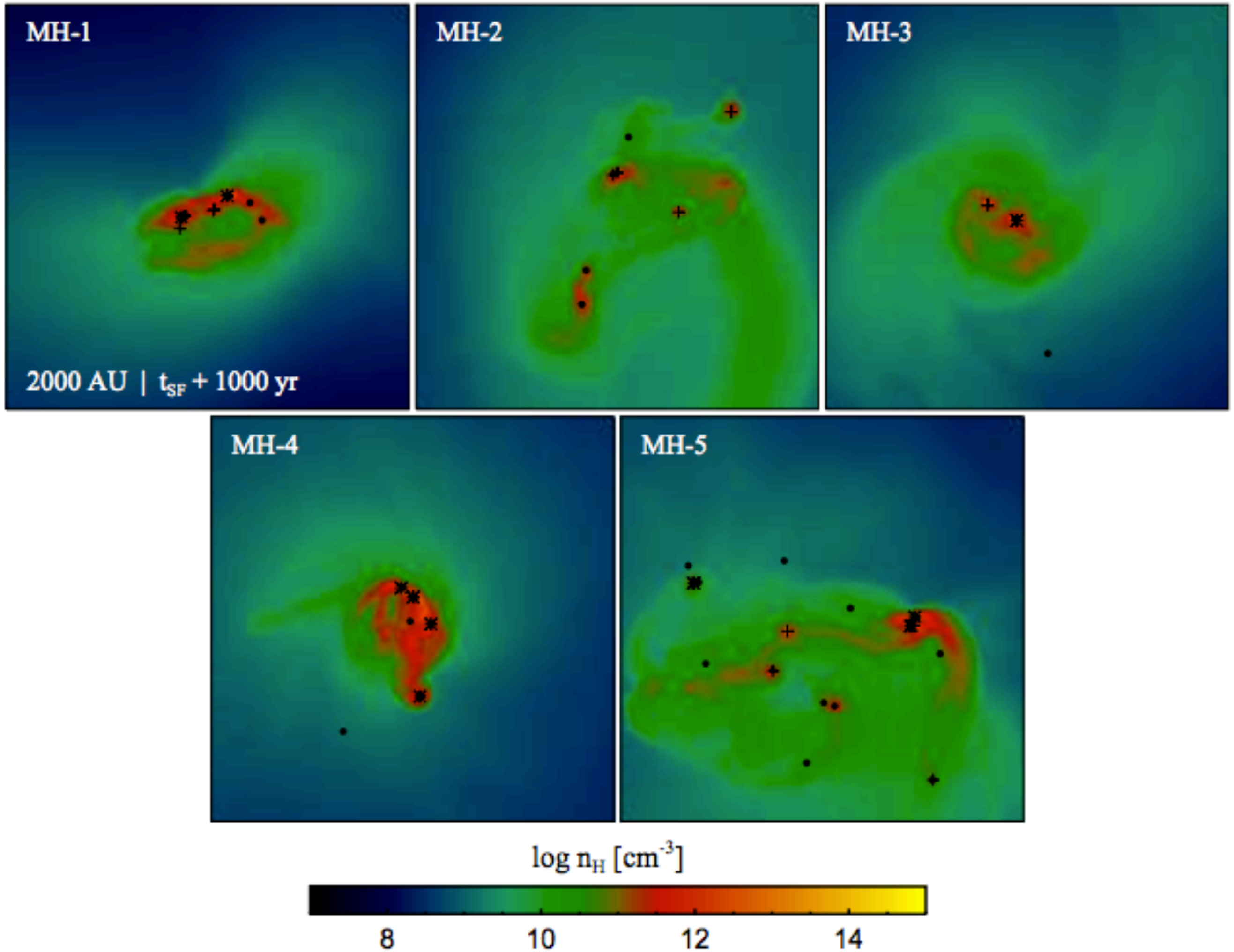
Right: the accretion rate through the disk and envelope as a function of radius from the central protostar helps to explain why the disk became so unstable. The accretion rate through the disk is considerably lower than the rate at which material is added to the disk. The system is unable to process the material in the disk quickly enough before more was added from the infalling envelope. As a result, the disk grows in mass, became gravitationally unstable, and ultimately fragments

Pop III Star Formation: further evolution (Greif et al. 2011)



- The panels show the density-squared weighted number density of H nuclei projected along the line of sight.
- The gas virializes on a scale of ~5 kpc, followed by the collapse of the central ~1 pc, where the gas becomes self-gravitating and decouples from the dark matter.
- In the final stages of the collapse, a fully molecular core on a scale of a few hundred AU forms.

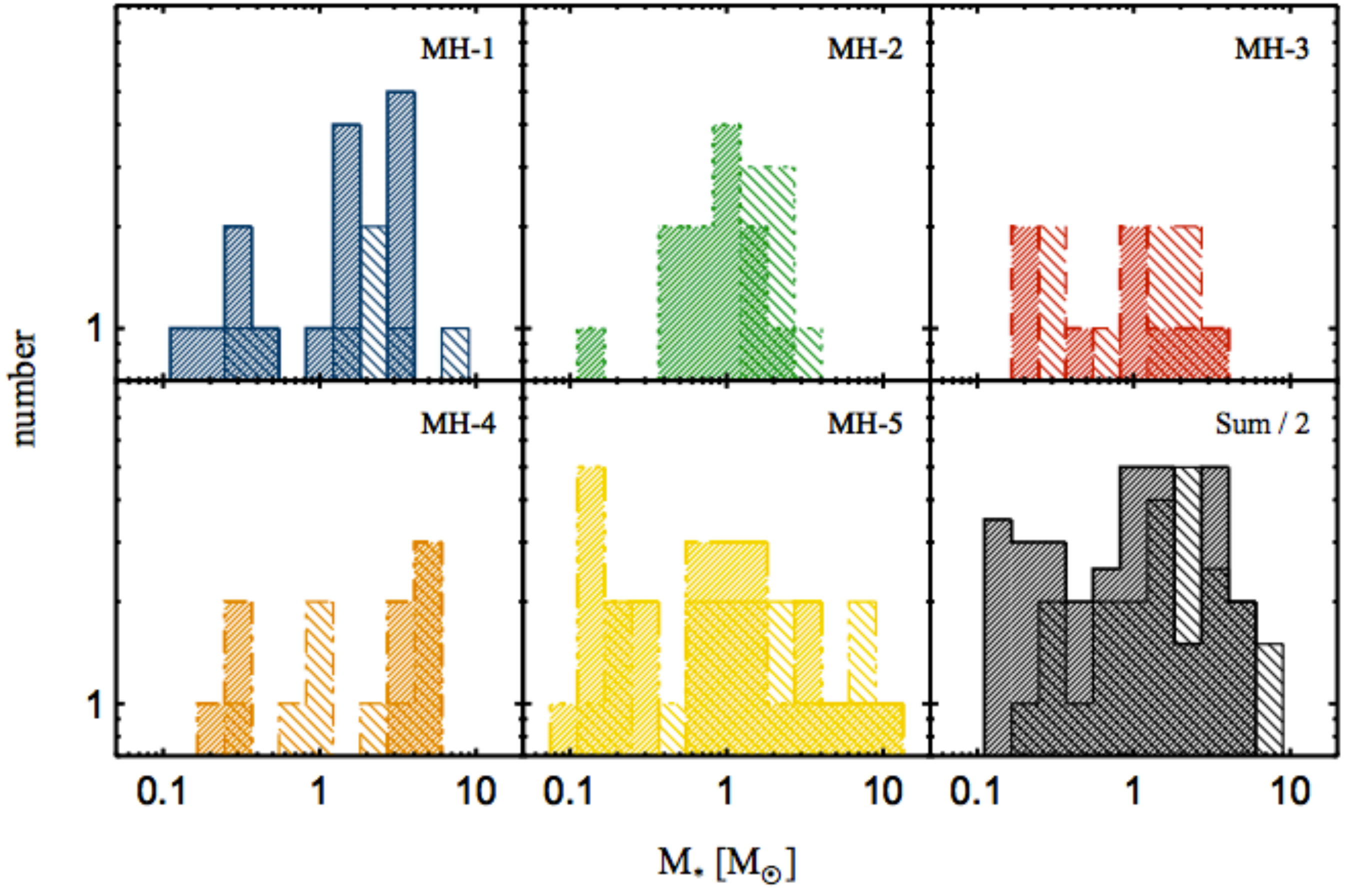
Pop III Star Formation: Further Evolution (Greif et al. 2011, ApJ, 737, 75)



- Again: gravitationally unstable disks -> multiple stars form
- Quite a bit of statistical variation

- The central 2000 AU after 1000 yr of fragmentation and accretion.
- Black dots, crosses and stars denote protostars with masses below 1 M☉, between 1 M☉ and 3 M☉, and above 3 M☉.
- A relatively rich protostellar cluster with a range of masses has survived in each case. In a few minihalos, low-mass protostars have been ejected out of the central gas cloud, such that they are no longer visible.
- In simulation MH-2, two independent clumps have collapsed almost simultaneously and formed their own clusters before eventually merging

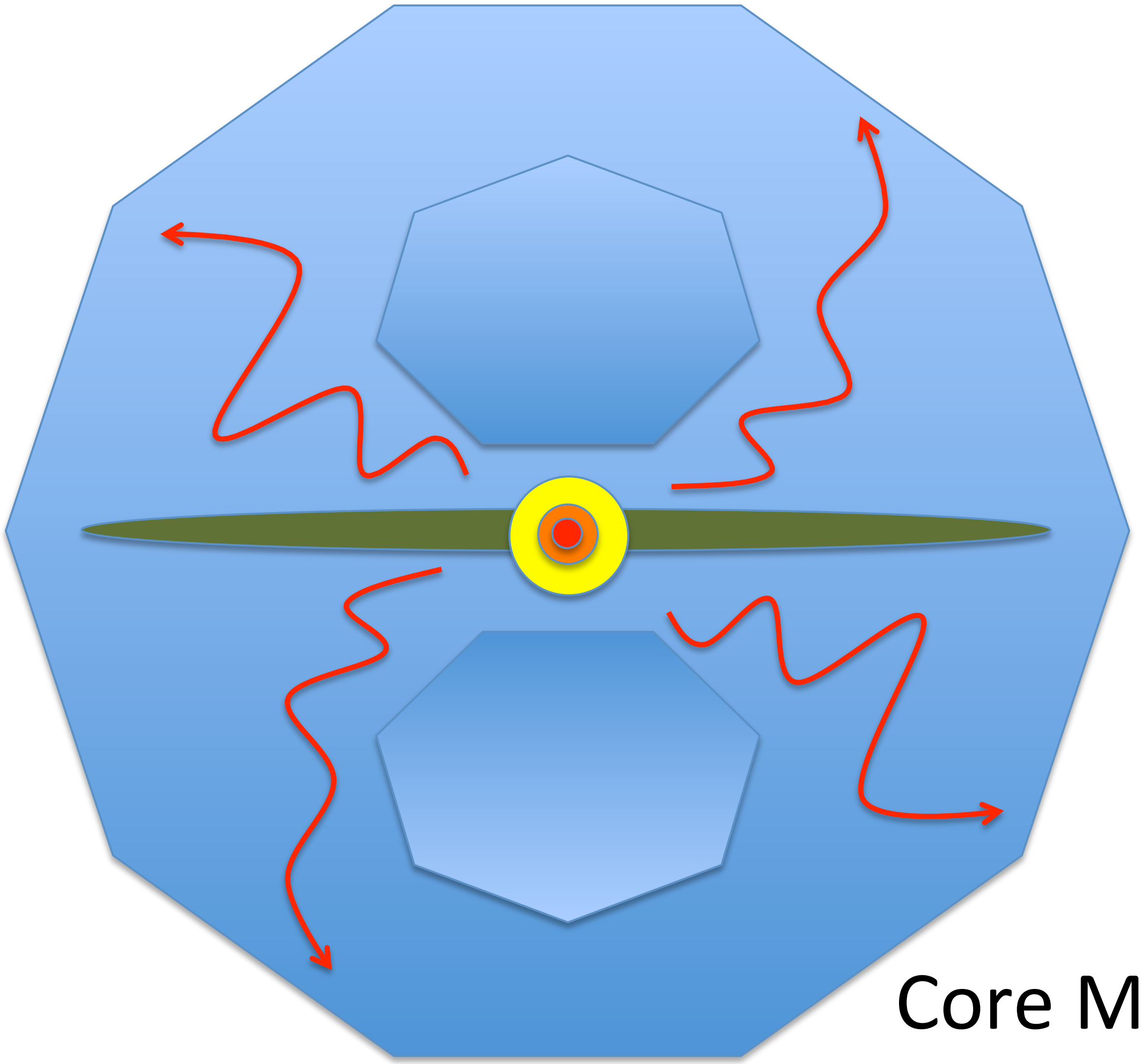
Pop III IMF: Early Situation (Greif et al. 2011, ApJ, 737, 75)



- After t ~ 1000 yr of fragmentation and accretion
- A "top-heavy" IMF: dN/dln m ~ m^x (x=0) (cf. Salpeter: x = -1.35)

Growth under Protostellar Feedback

- Radiation may impede growth, once M >10 Mo



Core Mass ~ M_J

Pop III Star Formation: Radiative Feedback (Stacy et al. arXiv: 1109.3147)

- A dominant binary has formed (~ 20 and ~10 $M_\odot$) after ~ 5 000 yr of accretion
- Note: Hosokawa et al. 2011 (Science Express): -> find similar result (only in 2D, but with better Radiation-Hydro)

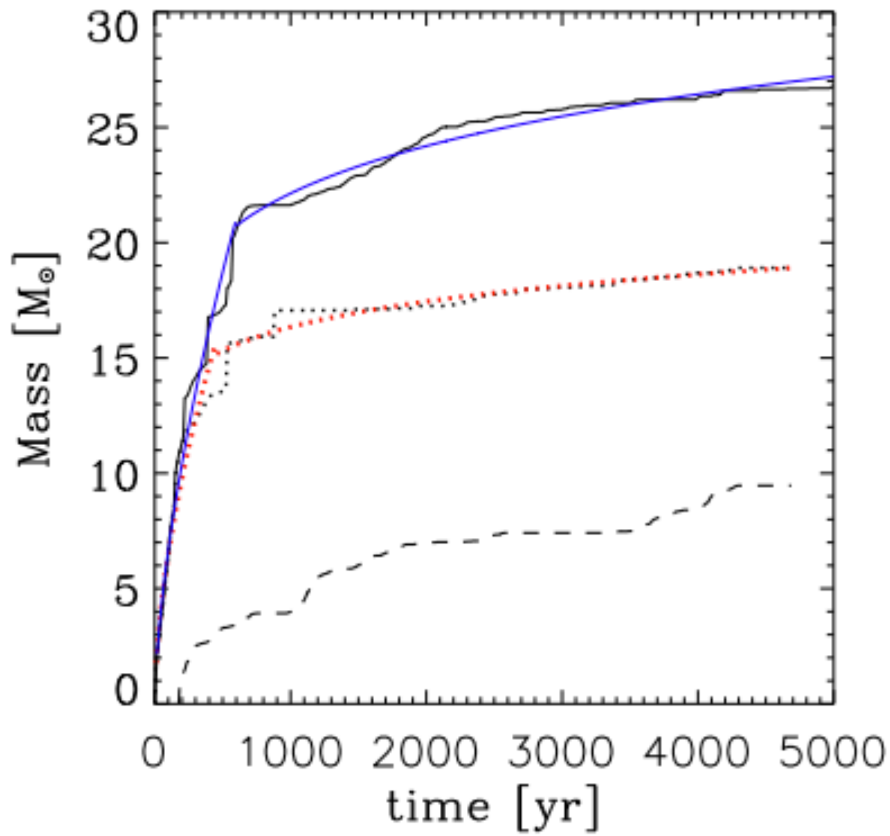


Figure 10. Effect of radiative feedback on protostellar accretion. Black solid line shows mass growth with no radiative feedback, while black dotted line shows the 'with-feedback' case. The blue solid line is a double powerlaw fit to the sink growth rate for the 'no-feedback' case, and the red dotted line is a double powerlaw fit for 'with-feedback' case. The dashed line shows the growth of the second-most massive sink in the 'with-feedback' case. Radiative feedback along with fragmentation-induced starvation leads to a divergence in the accretion histories in less than 1000 yr, and in the 'with-feedback' case the main sink does not grow beyond ~ 20 $M_\odot$ in the time of the simulation.

The first stars: final IMF

- Numerical simulations: (Bromm, Coppi, & Larson (1999, 2002), Abel, Bryan, & Norman (2000, 2002), Nakamura & Umemura (2001, 2002), Yoshida et al. (2006, 2008, O'Shea & Norman (2007), Gao et al. (2007), Clark et al. (2011), Greif et al. (2011)
- Likely Outcome -> **Top-heavy initial mass function (IMF)**